An improved direct extraction method for InP HBT small-signal model

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Abstract: In this paper, an improved direct extraction method to extract the model parameters in InP heterojunction bipolar transistor (HBT) small-signal equivalent circuit is presented and successfully applied to small-signal equivalent circuit of InP HBT. The distributed base-collector capacitance effect is taken into consideration in the adopted model. The extracting process of this method, which extracts parameters in turn from the peripheral parasitic elements to the intrinsic internal elements, is clearer than other direct extraction methods. Except for the parasitic parameters, all other parameters are calculated without any simplified approximation. This method relies on S parameters measurement. All of the equivalent circuit parameters are extracted directly from the S parameters without using approximations based on initial values. The direct extraction method is successfully validated on InP HBT in the frequency range of 0.1 ~ 40 GHz, and excellent agreement is achieved between the measured and calculated S parameters over the whole frequency range.

Key words: direct extraction method, InP HBT, small-signal model, parameter extraction
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一种改进的InP HBT小信号模型的直接提取法

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摘要: 提出了一种改进的直接提取方法来提取InP HBT小信号等效电路中的模型参数, 并将其成功地应用于InP异质结双极型晶体管 (HBT) 小信号等效电路。在所采用的模型中考虑了分布式基极-集电极电容效应。与其他直接参数提取方法相比, 该方法从外围寄生元件到内部本征元件依次进行参数提取, 提取过程较为清晰, 除寄生参数外, 其余所有的参数计算均未经过任何简化近似。该方法依赖于S参数的测量, 所有等效电路参数直接从S参数数据中提取, 而无需任何基于初始值的近似。在0.1 ~ 40 GHz的频率范围内, 直接提取法在InP HBT上得到了成功的验证, 并在整个频率范围内得到了较好的测量结果与计算结果的一致性。

关键词: 直接提取法; 磷化铟异质结双极型晶体管; 小信号模型; 参数提取
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Introduction

Owing to the characteristics of high-speed and high-frequency, InP HBT has been one of the most promising devices for future applications at microwave and millimeter-wave frequencies[1,3]. Small-signal model is the foot-stone of the entire transistor microwave model[4,5], and therefore accurate InP HBT device model is of great significance for the development of microwave and millimeter-wave integrated circuits.

With scaling down transistors’ size, in general, the structure of C-up HBT devices can ignore the extrinsic base-collector capacitance because the extrinsic area corresponding to the parasitic capacitance can be neglected[3]. Compact with the C-up devices, the E-up devices have a larger extrinsic area and need to consider the dis-
tributed base-collector capacitance effect. The effect is represented by extrinsic and intrinsic capacitances. By considering the effect, not only can this more clearly characterize the physical meaning of the base-collector capacitance, but it can further improve the model to make the model more accurate. In Ref. [6], although the authors have considered the effect in small-signal equivalent circuits, the calculation process is complicated and the calculation amount is too large. In Ref. [7, 8], the approximations used in the extraction process leads to inaccurate parameter extraction.

In recent years, many methods that extract small signal model parameters have been reported, mainly including direct extraction method [1-4, 6-8] and numerical optimization method [9, 10]. The numerical optimization method uses numerical methods to locate the optimal parameter values, and finally obtains simulated results with good fitting characteristics to the measured results. Nevertheless, the method depends on parameters’ initial value and may not even converge. With the direct extraction method, each parameter of the equivalent circuit could be extracted using equations. However, the disorganized extraction process makes the direct extraction method very computationally intensive and complicated. In Ref. [4, 6-7], although the authors used some simplified approximations, they also obtained more complex parameter expressions.

In order to overcome these difficulties, an improved direct extraction method for InP HBT small-signal model is proposed. This method in turn extracts the parameters of small-signal equivalent circuit from the peripheral elements to the internal elements. Compact with the other direct extraction methods, the method has clear extraction process, simple calculation of parameters, and few approximations calculation.

1 Small-signal equivalent circuit model

The adopted hybrid-π equivalent circuit for HBT small-signal modeling is shown in Fig. 1. This equivalent circuit includes two parts, i.e., the inner part contains intrinsic elements, and the outer part contains extrinsic elements [11]. In this model, $C_{be}$, $C_{bc}$, and $C_{ce}$ are pad parasitic capacitances, $L_b$, $L_c$ and $L_e$ are pad parasitic inductions, $R_b$, $R_c$, and $R_e$ are extrinsic resistances of collector, base, and emitter, respectively. These extrinsic elements are considered to be bias independent.

Intrinsic elements are supposed to be bias dependent, mainly including the dynamic base resistance $R_{ib}$, the dynamic base-emitter resistance $R_{be}$, the base-emitter capacitance $C_{be}$, the base-collector capacitance $C_{bc}$, the dynamic base-collector resistance $R_{bc}$, DC transconductance $g_m$ and delay time $\tau$. Besides, $C_{cs}$ is extrinsic base-collector capacitance, and it is considered to be bias independent.

2 Parameter extraction procedure

2.1 Extraction of parasitic parameters and the extrinsic resistances

Pad parasitic parameters consist of parasitic capaci-

\[ \begin{align*}
  Z_{11} &= R_b + R_{be} + \frac{R_{be}}{1 + g_m \cdot R_{be}}, \\
  Z_{12} &= R_e + \frac{R_{be}}{1 + g_m \cdot R_{be}}
\end{align*} \]

Fig. 1 HBT small-signal equivalent circuit

图1 HBT小信号等效电路

Fig. 2 open-collector equivalent circuit

图2 集电极开路等效电路

Fig. 3 Plots of the real part of $Z_{11}$, $Z_{12}$ and $Z_{22}$ versus $1/I_b$

图3 $Z_{11}$, $Z_{12}$ and $Z_{22}$的实部与$1/I_b$的关系图

The $Z$-parameters of the open-collector equivalent circuit in Fig. 2 is written as
When $I_b$ approaches $\infty$, $R_c$ and $R_e$ become very small at approximately 0 because the junction resistance and the junction current are inversely proportional. Moreover, with the increasing of the base current $I_b$, the total resistance of the base gradually approaches the base contact resistance \[^{15}\] i.e., $R_b + R_e = R_c$. Therefore, the intrinsic resistances $R_b$, $R_e$, and $R_c$ can be obtained (taking the real part of the $Z$ parameter to indicate the resistance value which makes the extraction result more accurate):

$$R_b = \text{real}(Z_{11} - Z_{13}),$$

$$R_e = \text{real}(Z_{13}),$$

$$R_c = \text{real}(Z_{22} - Z_{13}).$$

The relationship between \text{real}(Z_{11}, Z_{13}), \text{real}(Z_{22}, Z_{13}) and \text{real}(Z_{13}) and \text{Im}(Z_b)$ is linearly extrapolated to the ordinate to obtain the values of $R_b$, $R_e$, and $R_c$, as shown in Fig. 3\[^{15}\] . The extraction values of the bias-independent elements are tabulated in Table 1.

### Table 1 Extraction of extrinsic parameters values

<table>
<thead>
<tr>
<th>Parasitic parameters</th>
<th>$V_g=2.5,\text{V}, I_b=12.5,\text{mA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{bcx}/\text{fF}$</td>
<td>2.62</td>
</tr>
<tr>
<td>$C_{gc}/\text{fF}$</td>
<td>15.60</td>
</tr>
<tr>
<td>$C_{be}/\text{fF}$</td>
<td>16.20</td>
</tr>
<tr>
<td>$I_b/pA$</td>
<td>52.25</td>
</tr>
<tr>
<td>$I_{c}/pA$</td>
<td>57.75</td>
</tr>
<tr>
<td>$I_{g}/pA$</td>
<td>8.88</td>
</tr>
<tr>
<td>$R_b/\Omega$</td>
<td>4.27</td>
</tr>
<tr>
<td>$R_e/\Omega$</td>
<td>1.77</td>
</tr>
<tr>
<td>$R_c/\Omega$</td>
<td>7.31</td>
</tr>
</tbody>
</table>

### 2.2 Extraction of the extrinsic base–collector capacitance

Once all the parasitic elements are de-embedded, only the extrinsic base-collector capacitance $C_{bcx}$ and the intrinsic elements (inside the dashed line) are remained in the equivalent circuit, as shown in Fig. 4. To overcome the problem of unclear parameter extraction process, we need to extract sequentially from the external circuit to the internal circuit, that is, we need to extract $C_{bcx}$ first. However, the intrinsic equivalent circuit of the small signal equivalent circuit needs to be analyzed first before extracting the extrinsic base-collector capacitance. The $Z$-parameters of the equivalent circuit after de-embedding in Fig. 4, is written as

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{gmZ_b R_b + Z_{be} + R_e + \Delta Y_L}{A} & Z_{be} + \Delta Y_L \\ Z_{be} + \Delta Y_L - Z_{bcx} Z_{bcx} gm \end{bmatrix}$$

where $\Delta = Z_{11} + Z_{22} - Z_{12} - Z_{23} = \frac{Z_{bcx} + R_b}{1 + \text{Im}(Z_{bcx})}$, and

$$Z_{bcx} = Z_{11} - Z_{12} = \frac{1 + \text{Im}(Z_{bcx})}{Y_{bcx}(Z_{bcx} + R_b)}$$

The extrinsic base-collector capacitance can be expressed by taking the imaginary part of equation (11) equal to 0 as

$$C_{bcx} = \frac{1}{\omega \text{Im}(Z_3)} \left( \text{imag}(\Sigma Z) + \text{real}(Z_3) \right)$$

From (10), the dynamic base resistance $R_{bcx}$ can be expressed as

$$R_{bcx} = \frac{\left( Z_3 \right)}{1 - \text{Im}(\Sigma Z)},$$

where $\Sigma Z = Z_{11} + Z_{22} - Z_{12} - Z_{23} = \frac{Z_{bcx} + R_b}{1 + \text{Im}(Z_{bcx})}$. and

$$Z_{bcx} = Z_{11} - Z_{12} = \frac{1 + \text{Im}(Z_{bcx})}{Y_{bcx}(Z_{bcx} + R_b)}$$
Fig. 5 shows the extracted result as a function of frequency for \( \omega C_{\text{c}} \) at \( V_c = 2.5V \), \( I_c = 12.5 \) mA.

2.3 Extraction of the intrinsic elements

Once the parasitic elements and extrinsic base-collector capacitance is de-embedded, the remaining intrinsic elements of the small signal equivalent circuit model can be directly determined. The \( Z \)-parameter corresponding to the intrinsic circuit can be written as

\[
[Z \! \! \! \! \! \! _{\text{in}}] = \begin{pmatrix}
Z_{\text{m},11} & Z_{\text{m},12} \\
Z_{\text{m},21} & Z_{\text{m},22}
\end{pmatrix} = \begin{pmatrix}
R_{\text{be}} + \frac{Z_{\text{be}}}{1 + g_m Z_{\text{be}}} & \frac{Z_{\text{be}}}{1 + g_m Z_{\text{be}}} \\
Z_{\text{be}} - \frac{Z_{\text{be}}}{1 + g_m Z_{\text{be}}} & \frac{Z_{\text{be}}}{1 + g_m Z_{\text{be}}}
\end{pmatrix}
\]

From (13), the intrinsic base resistance \( R_{\text{be}} \) can be expressed as

\[
R_{\text{be}} = \text{real} \left( Z_{\text{m},11} - Z_{\text{m},12} \right)
\]

Fig. 6 shows the extracted result as a function of frequency for \( R_{\text{be}} \) at \( V_c = 2.5V \), \( I_c = 12.5 \) mA. Then de-embed off \( R_{\text{be}} \) and get a new small signal equivalent circuit, as shown in Fig. 7.

The \( Y \)-parameter of the corresponding equivalent circuit (Fig. 7) could be expressed as

\[
[Y \! \! \! \! \! \! _{\text{int}}] = \begin{pmatrix}
Y_{\text{m},11} & Y_{\text{m},12} \\
Y_{\text{m},21} & Y_{\text{m},22}
\end{pmatrix} = \begin{pmatrix}
\frac{1}{Z_{\text{be}}} & \frac{1}{Z_{\text{be}}} & \frac{-1}{Z_{\text{be}}} \\
\frac{1}{Z_{\text{be}}} + g_m & \frac{1}{Z_{\text{be}}}
\end{pmatrix}
\]

From (15), the intrinsic elements can be expressed as

\[
R_{\text{be}} = \frac{1}{\text{real} \left( Y_{\text{m},11} \right)}
\]

\[
C_{\text{be}} = \frac{\text{imag} \left( Y_{\text{m},11} \right)}{\omega}
\]

\[
R_{\text{be}} = \frac{1}{\text{real} \left( Y_{\text{m},11} + Y_{\text{m},12} \right)}
\]

\[
C_{\text{be}} = \frac{\text{imag} \left( Y_{\text{m},11} + Y_{\text{m},12} \right)}{\omega}
\]

\[
g_{\text{m0}} = \text{mag} \left( Y_{\text{m},12} \right)
\]

\[
\tau = \frac{-1}{\omega} \tan^{-1} \left( \frac{\text{imag} \left( Y_{\text{m},11} - Y_{\text{m},12} \right)}{\text{real} \left( Y_{\text{m},11} - Y_{\text{m},12} \right)} \right)
\]

3 Results and discussion

A hybrid-\( \pi \) small-signal equivalent circuit with distributed base-collector capacitance effect was adopted to study the microwave and millimeter-wave behavior of the InP HBT. An improved direct extraction method has been established to accurately extract the small-signal parameters. The improved method extracts parameters from the peripheral circuit to the internal circuit and gives a clearer solution process. Extraction results of small signal equivalent circuit are depicted in table 2, for bias points.

Table 2 Extraction of Intrinsic Parameters at \( V_c = 2.5V \)

<table>
<thead>
<tr>
<th>Intrinsic parameters</th>
<th>( I_c = 2.5 ) mA</th>
<th>( I_c = 12.5 ) mA</th>
<th>( I_c = 22 ) mA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{\text{iC}/F} )</td>
<td>49.21</td>
<td>35.60</td>
<td>58.31</td>
</tr>
<tr>
<td>( R_{\text{iE}/\Omega} )</td>
<td>36.22</td>
<td>8.32</td>
<td>2.45</td>
</tr>
<tr>
<td>( R_{\text{iE}/\Omega} )</td>
<td>2497.68</td>
<td>121.02</td>
<td>113.50</td>
</tr>
<tr>
<td>( R_{\text{iE}/\Omega} )</td>
<td>1.90×10⁴</td>
<td>2.66×10⁴</td>
<td>1.04×10⁴</td>
</tr>
<tr>
<td>( C_{\text{be}/F} )</td>
<td>32.03</td>
<td>18.26</td>
<td>16.59</td>
</tr>
<tr>
<td>( C_{\text{be}/F} )</td>
<td>145.30</td>
<td>718.85</td>
<td>915.30</td>
</tr>
<tr>
<td>( g_{\text{mE}/S} )</td>
<td>0.08</td>
<td>0.60</td>
<td>1.19</td>
</tr>
</tbody>
</table>

Fig. 8 shows the calculated \( S \)-parameters of the small-signal equivalent circuit of the HBT including the distributed base-collector capacitance with the measured data. The comparison shown in Fig. 8 demonstrates a good agreement from 0.1 ~ 40.0 GHz, which also verifies the validity of the model and extraction techniques.

However, the Smith plots of \( S \) parameters do not clearly reflect agreement of fit between measured and calculated data. The residual error between the measured
results and the calculated results are quantified using the following equation [15]:

$$ E = \frac{1}{4N} \sum_{i=1}^{N} \sum_{j=1}^{N} \left| S_{ij}^{m}(f_i) - S_{ij}^{c}(f_i) \right| $$

where $N$ is the number of frequency points, $S_{ij}^{m}(f_i)$ and $S_{ij}^{c}(f_i)$ are the measured and calculated S-parameters at frequency $f_i$, respectively. The residual errors between the measured and modeled S-parameters are around 3, 3–3, 8%.

4 Conclusion

An improved direct extraction method for the hybrid-$\pi$ small-signal equivalent circuit with the base-collector capacitance effect has been proposed. The extracting process of this method, which extracts parameters in turn from the peripheral parasitic elements to the intrinsic internal elements, is clearer than other direct extraction methods. Furthermore, this method can extract all intrinsic parameters directly by the equation without approximation and numerical optimization. Good agreement is obtained between calculated and measured results for an InP HBT with 1×15 $\mu$m$^2$ emitter area over a wide range of bias points up to 40GHz.

References


