Design of a $\text{TE}_{34,10}$ mode cylindrical cavity for MW level gyrotron

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Abstract: A cylindrical gyrotron cavity is designed and optimized with a high order mode $\text{TE}_{34,10}$ to deliver an output power of megawatts level at 140 GHz. Analysis on mode competition indicates that the two adjacent modes $\text{TE}_{33,10}$ and $\text{TE}_{31,11 +}$ involve in competition and cause significant decreasing of output power of $\text{TE}_{34,10}$. To suppress the competition hysteresis loops of $\text{TE}_{31,11 +}$ and $\text{TE}_{33,10}$ with $\text{TE}_{34,10}$ are calculated, which indicates $\text{TE}_{34,10}$ can inhibit growth of the other two modes with decreasing magnetic field from its low efficiency single mode oscillation zone while it turned out just the opposite with increasing magnetic field due to the earlier oscillation of the competitors. Based on the results, a multimode time-dependent calculation including 42 modes is carried out with magnetic field dropping from 5.59 T to 5.51 T, the results show that mode competition is successfully suppressed, the operation mode $\text{TE}_{34,10}$ realizes stable single mode oscillation with an output power of 0.96 MW and an electron efficiency of 36.7% at 140 GHz.

Key words: gyrotron; cylindrical cavities; mode competition

PACS: 84.40.Ik, 84.40.Fe, 07.57.Hm

Introduction

Gyrotrons can produce hundreds of kilowatts or higher continuous-wave (CW) power in millimeter wave and higher frequency bands\[1-2\]. They have important applications in magnetically confined fusion\[3-6\] such as plasma startup, heating, current driving and diagnostics\[7-14\]. To handle an output power of MW level the operating mode order should be increased as high as possible to minimize the heat density on the cavity wall but the mode competition deteriorates rapidly which restricts the further increasing of the mode order. In gyrotrons with cylindrical cavities which have realized MW level...
output power high order modes such as $TE_{2530}$ and $TE_{2550}$ have been successfully used and achieved continuous operation time of hundreds of seconds while higher order modes such as $TE_{1113}$ and $TE_{1115}$ and so on have been studied with numerical calculations or short-duration experiments to further reduce heat dissipation difficulty. However, stable operation of the higher order modes is always threatened by the mode competition. In the modes mentioned above $TE_{1113}$ is a promising candidate for MW level long pulse gyrotrons. Cavity designs with $TE_{1113}$ have been carried out with PIC simulations in Ref. [12] and with a multimode time-dependent code named SELFIT in Ref. [14] which show the $TE_{1113}$ cavity have sufficient power capacity and acceptable efficiency for long pulse gyrotrons around 170 GHz.

In this paper a cylindrical $TE_{1113}$ cavity is designed with an in-house developed code GYO. We mainly focus on the possible competitors to the $TE_{1113}$ mode and try to avoid the interference in the steady operation. A time-varying magnetic field is introduced into the starting process to realize single mode stable oscillation. The RF losses in the cavity is calculated which is far less than the critical value. The structure of the article is organized as below: the first part briefly reviews the theoretical models used in GYO which consists of LN-module for linear theoretical calculation, SSC-module for single-mode self-consistent theoretical calculations and MTC-module for multimode time-dependent theoretical calculations. The second part presents the design of the cavity and suppression of competition modes. In the third part the loss power density on the cavity wall is obtained. Conclusion will be given in the end.

1 Theoretical model

The theoretical models used in GYO consist of the linear theory, the nonlinear self-consistent theory and the multimode time-dependent theory which correspond to the LN module, SSC module and MTC module. The linear theory is always used to obtain the oscillation starting conditions and the nonlinear self-consistent theory represents the stationary interaction of a single mode and the beam which could be used to optimize the parameters. The multimode time-dependent theory can include several modes in the calculation and acquire multimode interaction state in time domain which is helpful in analysis of mode competition.

One of the most important parameters derived from the linear theory is the threshold current which could be described as [15]

$$-1/I_{th} = \frac{C_m}{S \gamma_{mc} \lambda (\pi \frac{\tau}{2} - 1)} \left| f(z) \right|^2 \left| dz \right|^{-1} \cdot \left( \frac{k_m C_m}{\beta_{0z}} \right)^2 \left( \frac{c_m \gamma_{mc} \beta_{0z}}{2 \Omega x} \right)^{2(1-i)} \cdot \left( s + \frac{1}{2} \frac{2 \omega \beta_{0z}}{\nu_{th}} \right) \cdot \left| \int_0^{\pi} f(z) e^{i \phi} \right|^2 \cdot \left( \frac{\nu_{th}}{d_{0z}} \right)^{-2}$$

(1)

where $C_m = \left\{ \sqrt{\frac{\pi (x_m - m \lambda)}{S \gamma_{mc} \lambda \tau (f(z))}} \right\} \left| f(z) \right|^2 \left| dz \right|^{-1} \cdot \left( \frac{\nu_{th}}{d_{0z}} \right)^{-2}$

$$G_m = J m \pm \pi (k m \gamma_{mc} \beta_{0z})$$

(2)

where $f = \left| f(z) \right|^2$ is field profile function $Q$ is quality factor of the cavity $Z_0$ is wave impedance in vacuum $\eta$ is charge-mass ratio $s$ is the harmonic number $m + s \pi$ corresponds to the case of azimuthal co-rotation and counter-rotation of the wave and the electron $\beta_{0z}$ is transverse velocity and $\beta_{0x}$ is axial velocity of electrons normalized by light speed $c$ respectively $k_{mz}$ is transverse wave number $R_e$ is injection radius of the beam $\Delta (z)$ is the mode detuning parameter given by

$$\Delta (z) = \frac{\omega}{\nu_{th}} (1 - \frac{\pi \Omega_0}{\omega \gamma_{mc}}) \cdot \frac{\gamma_0}{\gamma_{mc}}$$

(4)

where $\omega$ is frequency of wave and $\Omega_0 = eB/m_s$ is non-relativistic cyclotron frequency of electron. The coupling coefficient represents the coupling strength between electron beam and specific mode and could be described by [16]

$$G_B = \left( \frac{x_{mz}}{m \lambda} \right) \left( s + \frac{1}{2} \right) \left( \frac{\nu_{th}}{d_{0z}} \right)^{-2} \cdot \left( \frac{\nu_{th}}{d_{0z}} \right)^{-2}$$

(5)

Equation (5) indicates the strongest coupling occurs at the first maximum of $J_{n-1}(k m \gamma_{mc} \beta_{0z})$ which is also the first zero of $J_{n+1}(k m \gamma_{mc} \beta_{0z})$.

The nonlinear self-consistent theory is composed of the adiabatic equation for electron motion and the wave equation for the RF field profile function which can be expressed respectively as [17]

$$\frac{d}{dt} f(z) + i \gamma_0 \gamma_{mc} \frac{1}{\gamma_0} \left( \frac{1}{\gamma_0} \frac{d}{dt} f(z) \right) = - \left( \frac{1}{2} \frac{\omega}{\nu_{th}} \gamma_0 \gamma_{mc} \frac{1}{\gamma_0} \right) \left( \frac{1}{\gamma_0} \frac{d}{dt} f(z) \right) \left( \frac{1}{\gamma_0} \frac{d}{dt} f(z) \right)$$

(6)

and

$$\frac{d^2}{dt^2} f(z) + \frac{\nu_{th}}{\gamma_0} (1 - \beta_{0z}^2 - \gamma_0^2) \frac{d}{dt} f(z) = - \left( \frac{\omega}{\gamma_0} \gamma_{mc} \beta_{0z} \right) \left( \frac{1}{\gamma_0} \frac{d}{dt} f(z) \right) \left( \frac{1}{\gamma_0} \frac{d}{dt} f(z) \right)$$

(7)

where $P = \nu_{th} \exp (-i \lambda)$ is normalized momentum variable $\lambda = \nu_{th} / c$ is the normalized transverse momentum $\lambda_{d0} = \nu_{th} / c$ is normalized initial axial velocity $\lambda = \left( \frac{\omega}{\nu_{th}} - \Omega \right)$ $\lambda_{d0} = \nu_{th} / \nu_{d0}$ is normalized time simultaneous variable $\Omega = \Omega \lambda_{d0} / c$ is also normalized cyclotron frequency of electron $\nu_{d0}$ is an arbitrary radius for normalization $\nu_{d0}$ is the moment when electrons enter the cavity $\lambda = t - \nu_{th} \lambda_{d0} \phi$ is the polar angle of guiding center $F = C_m x_{mz} \gamma_{mc} \eta / c^2 \times 2^{-1} - 1$ is normalized field profile function $C_m = \sqrt{\frac{\pi (x_m - m \lambda)}{S \gamma_{mc} \lambda}}$ is a normalization constant $\nu_{mc} = \nu_{th} / c$ is normalized wave guide radius $\nu_{mc} = \nu_{th} / c$ is normalized injection radius $k_m = k m / \nu_{th} \nu_{d0}$ is normalized transverse wave number $\nu_{d0} = \nu_{th} / c$ is the wave period $\nu_{d0}$ which are given by $[18]$.

$$\frac{d \nu_{d0}}{dt} + \frac{\nu_{d0} \nu_{d0} / \gamma_0}{2 \nu_{th}} = \frac{-1}{\nu_{th} \gamma_0} \Im \hat{R}(t)$$

(8)
Design of the cavity and realization of single-mode steady operation

A simple three-segment structure including down taper uniform section and up taper is adopted in the cavity. The length of the uniform section is expressed as \( L \) and the radius of the uniform section is \( R_0 \). \( R_0 \) is set as 25.45 mm according to the cutoff frequency of the waveguide. The cavity structure and field distribution under different \( L \) are shown in Fig. 1(a). The resonant frequency and the \( Q \)-factor of the cavity are shown in Fig. 1(b).

![Cavity structure and field distribution](image)

**Fig. 1** (a) Cavity structure and field distribution (b) resonant frequency and \( Q \)-factor of the cavity

We choose co-rotation of the wave and the beam for better beam-wave interaction which means the operation mode is \( \text{TE}_{34\beta_0} \). With beam voltage 75 kV, beam current 35 A, pitch factor 1.5 and injection radius 12.16 mm, the output power of the cavity is calculated with SSC module, the results are shown in Fig. 2. As shown in the figure, the output power rises with \( L \) but the growth rate slows down when it reaches 16 mm. Although longer cavity length can increase output power slightly, it will bring the risk of deterioration of mode competition, so we choose \( L \) to be 16 mm, the maximum output power could reach 1.1 MW at magnetic field of 5.52 T with an efficiency of 41.9%.

![Output power versus magnetic field](image)

**Fig. 2** Output power versus magnetic field under different \( L \) calculated by SSC module

Now we consider the modes which might participate in competition with operation mode \( \text{TE}_{34\beta_0} \). The coupling coefficients and corresponding eigenvalues of the nearby modes are plotted in Fig. 3(a). In which \( \text{TE}_{34\beta_1} \), \( \text{TE}_{34\beta_2} \), and \( \text{TE}_{33\beta_6} \) have the coupling coefficient similar to \( \text{TE}_{34\beta_0} \). The threshold currents of the modes are plotted in Fig. 3(b) where we can see that many of the neighbor modes have starting current below 35 A and therefore are capable of oscillating so the process of starting oscillation is also a process of competition among these modes.

The MTC module is used to study the starting process of the oscillation of 13 modes thought to be most dangerous are included according to Fig. 3(a) and fixed static magnetic field is adopted. The results are counted in Fig. 4(a) which shows that \( \text{TE}_{34\beta_0} \) can only achieve stable oscillation between 5.57 T and 5.68 T but fails at magnetic field between 5.44 T and 5.56 T where \( \text{TE}_{34\beta_0} \) or \( \text{TE}_{34\beta_1} \) dominates the oscillation. The results of SSC calculation of the three modes are also given in Fig. 4(a) which expect an output power of 1.1 MW for \( \text{TE}_{34\beta_0} \) but only 664 kW could be achieved according to the MTC calculation due to the competition of \( \text{TE}_{34\beta_0} \) and \( \text{TE}_{34\beta_1} \).

**Fig. 4** (b) shows the time behavior of the modes with a fixed magnetic field of 5.55 T, the initial power of each mode is set to be 0.1 kW. The power of \( \text{TE}_{34\beta_2} \)
The magnetic field we use in the calculations above is a fixed one which doesn’t seem to achieve an efficient operation state for TE_{33\parallel} mode. From Fig. 4(a) we can see that TE_{33\parallel} can obtain stable oscillation at magnetic field between 5.57 T and 5.68 T due to the inhibition effect of the preset field on the other mode a time-varying magnetic field which decreases after the operation mode achieving stable oscillation might be helpful to improve the efficiency. To verify the feasibility of the method it is necessary to obtain the hysteresis curves of TE_{31\parallel}, TE_{33\parallel} with TE_{33\parallel} first. The scanning range of the magnetic field is chosen to be 5.45 T ~ 5.59 T the scanning process is that the magnetic field decreases from 5.59 T to 5.45 T at a uniform speed in 500 ns then maintains 5.45 T for 100 ns and then rises to 5.59 T again in 500 ns. The calculated hysteresis loop of TE_{33\parallel} and TE_{33\parallel} is shown in Fig. 5(a). As shown in the Fig. 5(b) TE_{33\parallel} starts oscillation first at B = 5.59 T while TE_{33\parallel} damps rapidly. In the process of magnetic field falling from 5.59 T TE_{33\parallel} remains dominant and its output power increases gradually and reaches the maximum at about 5.5 T. After that the output power of TE_{33\parallel} begins to decrease as the magnetic field decreases. When the magnetic field reaches 5.485 T the dominant mode is switched from TE_{33\parallel} to TE_{33\parallel}. The output power of TE_{33\parallel} decreases rapidly while TE_{33\parallel} rises to the peak. As the magnetic field continues to decrease the power of TE_{33\parallel} is still rising until it reaches 5.45 T. After the retention time of 100 ns the magnetic field begins to increase TE_{33\parallel} remains dominant until the magnetic field increases to 5.58 T where the oscillation mode is switched from TE_{33\parallel} to TE_{33\parallel} again. In the magnetic field area between 5.485 T and 5.58 T both modes can achieve steady single-mode oscillation the dominant mode is determined by the changing direction of the magnetic field. Similar conclusion can also be obtained from...
The hysteresis loop between $TE_{3431}$ and $TE_{3430}$, as shown in Fig. 5(b). In fact, the above phenomenon is caused by the inhibition effect of the mode which obtains a certain oscillation amplitude first on the other modes. So, to realize high-power output of $TE_{3430}$, we can start at arbitrary magnetic field between 5.57 T and 5.68 T where $TE_{3430}$ can retain single-mode steady oscillation[1] and change the magnetic field to a more efficient point after the expected mode is excited.

![Fig. 5](image)

Fig. 5 (a) Hysteresis loop between $TE_{3430}$ and $TE_{3431}$. (b) Hysteresis loop between $TE_{3430}$ and $TE_{3431}$. The magnetic field varies as: decreasing from 5.59 T to 5.45 T in 500 ns then maintaining 5.45 T for 100 ns and then rising to 5.59 T again in 500 ns.

The dependence of magnetic field on time we use finally is shown in Fig. 6 which also contains the corresponding MTC calculation results including 42 neighbor modes with time-varying magnetic field.

![Fig. 6](image)

Fig. 6 MTC calculation results including 42 modes with time-varying magnetic field.

The ohmic wall loss can be calculated as below[20]

$$\frac{dP_{\text{loss}}}{dt} = \frac{\delta}{4\pi\mu_0}\left[\frac{1}{x_{mw}^2} - m^2\right]\left(k_{mn}^2 + f_1^2 + \frac{m^2}{\omega^2}\right)$$

where $\delta = \sqrt{2/\mu_0}\mu_0$ is the skin depth, $\mu_0$ is the permeability, $\sigma$ is the electrical conductivity. The other parameters are the same as before.

The wall loading of the cavity is also shown in Fig. 7, the peak loss power density is about 0.67 kW/cm² with an output power of about 1 MW which is far less than the critical value. It makes the heat dissipation easier and is helpful to improve the stability and reliability of the gyrotron.

4 Conclusions

A cavity operating at $TE_{3430}$ is designed in this article. According to the analysis of eigenvalues and coupling coefficients of the neighbor modes, $TE_{3430}$ and $TE_{3431}$ are more dangerous to the operation mode than the else which is verified by MTC calculations. Using the time-varying magnetic field method the mode competitive power of 964 kW and an efficiency of 36.7% at magnetic field of 5.51 T. The mode competition is successfully suppressed[1] and the power of all other modes is far less than 1 kW at the end of the calculation. The output power of $TE_{3430}$ is still smaller than the maximum 1.1 MW obtained by SSC calculation[1] which may be caused by the influence of the other modes with small amplitudes on the interaction between the operation mode and the beam.
tition is suppressed and single-mode steady operation with an output power of 964 kW at 140 GHz is achieved. The peak ohmic wall loading of the cavity is only 0.67 kW/cm² which is beneficial for cooling system design and achieving stable CW operation. Besides, according to our analysis, so adjustment methods based on other parameters may be also feasible such as the beam voltage, pitch factor, etc.

References