大功率毫米波环板行波管注波互作用线性理论研究*

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摘要 考虑了电子注的厚度和相对论效应，采用场匹配和变分法中的特殊方法—Ritz-Galerkin 法，导出了考虑电子束空间电荷效应的环板行波管的热色散方程，并对该方程进行数值求解，得到了其小讯号增益，讨论了慢波系统几何尺寸和电子注参量对小信号增益的影响，为毫米波环板行波管的设计提供了理论依据.
关键词 环板行波管，小讯号增益，线性理论.

LINEAR THEORY OF THE BEAM-WAVE INTERACTION IN THE HIGH POWER RING-PLANE TWT AT THE MILLIMETER WAVE BAND*

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Abstract In consideration of the thickness and the relativistic effect of the hollow electron beam, the hot dispersion relation including the electron beam space charge effect for the ring-plane traveling wave tube was derived by making use of the Ritz-Galerkin method and combining with the field matching method. The numerical results given were in terms of the small signal gain curve. Finally, the influences of the radius and current of the electron beam, the acceleration voltage and the geometrical dimensions of the slow-wave structure on the small signal gain were discussed. The results presented in this paper provide theoretical basis on designing the millimeter wave ring-plane traveling wave tube.
Key words ring-plane TWT, small signal gain, linear theory.

引言

环板行波管由于具有大尺寸、高输出功率等特点，在毫米波段有很好的应用前景. C. K. Birdsall 等最早对其进行了实验研究[1]，并在 9 mm 波段成功研制工作电压为 100 kV，输出功率为 40 kW 的样管[2]，用自洽线性理论研究环板行波管的注波互作用未见报导.

文献[3]中，H. P. Freund 等给出了无限薄电子束螺旋线行波管的自洽场理论；文献[4]中，给出了无限薄电子束脊加载环板行波管的自洽场理论. 上述文献虽然在理论上得到了考虑空间电荷效应的热色散方程，但无限薄电子束模型在现实中是无法实现的，实际上，大多数行波管都是采用实心电子束或空心电子束，电子束总是有一定厚度的. 为了更准确地反映环板行波管注波互作用特性，本文针对实际的环板行波管，采用动力学理论研究了相对论空心电子束与慢电磁导波的相互作用，给出了其热色散方程，并对其进行了数值分析. 结果表明：对应最大增益，有一饱和注电压存在；小信号增益随电子注电流增加而增加；电子注半径对小信号增益影响强烈，小信号增益随着电子注外半径的增大而增大. 与文献[4]比较，环板行波管增益更高，但其带宽相对较窄，是一种毫米波段的高增益器件.

1 场的表达式

如图 1 所示，在环板慢波结构中引入一个注电流为 I_0，以速度 v_0 沿轴向运动，内半径为 a，外半径

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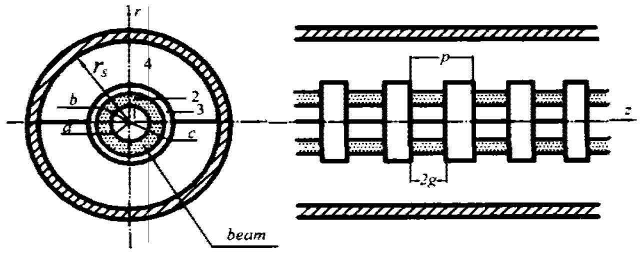

图1 具有相对论空心电子注的环板慢波系统示意图
Fig. 1 Schematic Diagram of the ring-plane slow wave structure with relavitistic hollow electron beam

为 b 的电子注.假设环的半径为 c,节距为 p,间隙为 $2g$,屏蔽筒半径为 r_s.现在,将整个结构分为4个区,其中2区为电子束区域,该区慢电磁导波场的纵向分量 E_{z2}、H_{z2}满足波动方程

$$\begin{cases}\nabla_\perp^2 E_{z2}+\gamma_s'^2 E_{z2}=0,\\ \nabla_\perp^2 H_{z2}+\gamma_s'^2 H_{z2}=0,\end{cases} \tag{1}$$

其中 $\gamma_s'^2=(\beta_s^2-k^2)\left[1-\frac{\omega_p^2}{(\omega-\beta_s\upsilon_0)^2}\right]$,$\omega_p$ 为束电子的等离子体频率,$\beta_s=\beta_0+2\pi s/p$,为第 s 次空间谐波的纵向传播常数,$k^2=\omega^2\mu_0\varepsilon_0$,$k$ 为自由空间波数.

其它各区慢电磁波场的纵向分量 E_z、H_z 满足如下波动方程

$$\begin{cases}\nabla_\perp^2 E_z+\gamma_s^2 E_z=0,\\ \nabla_\perp^2 H_z+\gamma_s^2 H_{z2}=0,\end{cases} \tag{2}$$

其中 $\gamma_s^2=\beta_s^2-k^2$.

由于对称模式的场对注波互作用是有益的[1],所以这里只讨论对称模式场的分布.下面给出各区纵向场的分布,其余场分量可由麦克斯韦方程导出.

1区($r\leqslant a$):

$$E_{z1}(r,\theta,z)=\sum_{l=0,2,4\ldots}^{+\infty}\sum_{s=-\infty}^{+\infty}A_{ls}^1 I_l(\gamma_s r)\cos(l\theta)e^{-j\beta_s z}, \tag{3}$$

$$H_{z1}(r,\theta,z)=\sum_{l=2,4}^{+\infty}\sum_{s=-\infty}^{+\infty}B_{ls}^1 I_l(\gamma_s r)\sin(l\theta)e^{-j\beta_s z}, \tag{4}$$

2区($a<r\leqslant b$):

$$E_{z2}(r,\theta,z)=\sum_{l=0,2,4\ldots}^{+\infty}\sum_{s=-\infty}^{+\infty}[A_{ls}^2 I_l(\gamma_s' r)+C_{ls}^2 K_l(\gamma_s' r)]\cos(l\theta)e^{-j\beta_s z}, \tag{5}$$

$$H_{z2}(r,\theta,z)=\sum_{l=2,4,6}^{+\infty}\sum_{s=-\infty}^{+\infty}[B_{ls}^2 I_l(\gamma_s' r)+D_{ls}^2 K_l(\gamma_s' r)]\sin(l\theta)e^{-j\beta_s z}, \tag{6}$$

3区($b<r\leqslant c$):

$$E_{z3}(r,\theta,z)=\sum_{l=0,2,4}^{+\infty}\sum_{s=-\infty}^{+\infty}[A_{ls}^3 I_l(\gamma_s r)+C_{ls}^3 K_l(\gamma_s r)]\cos(l\theta)e^{-j\beta_s z}, \tag{7}$$

$$H_{z3}(r,\theta,z)=\sum_{l=2,4,6}^{+\infty}\sum_{s=-\infty}^{+\infty}[B_{ls}^3 I_l(\gamma_s r)+D_{ls}^3 K_l(\gamma_s r)]\sin(l\theta)e^{-j\beta_s z}, \tag{8}$$

上述三个区的场表达式有10个未知幅值系数,利用 $r=a$,及 $r=b$,即电子束表面切向场分量连续的边界条件,消去8个变量;然后,假设3区切向电场分量在环面上($r=c$)的幅值分别为 F_{ls}、H_{ls},经过繁杂的运算,可得用 F_{ls}、H_{ls}表达的3区场表达式

$$E_{z2}(r,\theta,z)=\sum_{l=0,2,4\ldots}^{+\infty}\sum_{s=-\infty}^{+\infty}\left\{F_{ls}\left[\frac{Q_2(W_1I_l(\gamma_s r)+W_3K_l(\gamma_s r))}{P_1Q_2-P_2Q_1}+\frac{Q_1(W_2I_l(\gamma_s r)+W_4K_l(\gamma_s r))}{P_2Q_1-P_1Q_2}\right]+H_{ls}\left[\frac{P_2(W_1I_l(\gamma_s r)+W_3K_l(\gamma_s r))}{P_1Q_2-P_2Q_1}+\frac{P_1(W_2I_l(\gamma_s r)+W_4K_l(\gamma_s r))}{P_2Q_1-P_1Q_2}\right]\right\}\cos(l\theta)e^{-j\beta_s z}, \tag{9}$$

$$E_{z3}(r,\theta,z)=\sum_{l=2,4\ldots}^{+\infty}\sum_{s=-\infty}^{+\infty}\left\{F_{ls}\left[\frac{Q_2(W_5I_l(\gamma_s r)+W_7K_l(\gamma_s r))}{P_1Q_2-P_2Q_1}+\frac{Q_1(W_6I_l(\gamma_s r)+W_8K_l(\gamma_s r))}{P_2Q_1-P_1Q_2}\right]+H_{ls}\left[\frac{P_2(W_5I_l(\gamma_s r)+W_7K_l(\gamma_s r))}{P_1Q_2-P_2Q_1}+\frac{P_1(W_6I_l(\gamma_s r)+W_8K_l(\gamma_s r))}{P_2Q_1-P_1Q_2}\right]\right\}\sin(l\theta)e^{-j\beta_s z}, \tag{10}$$

其中

$$P_1=W_1I_l(\gamma_s c)+W_3K_l(\gamma_s c),$$
$$P_2=W_2I_l(\gamma_s c)+W_4K_l(\gamma_s c),$$
$$Q_1=\frac{W_1I_l(\gamma_s c)\beta_s l}{\gamma_s^2 c}+\frac{W_5I'_l(\gamma_s c)\omega\mu_0}{\gamma_s}+\frac{W_3K_l(\gamma_s c)\beta_s l}{\gamma_s^2 c}+\frac{W_7K'_l(\gamma_s c)\omega\mu_0}{\gamma_s},$$
$$Q_2=\frac{W_2I_l(\gamma_s c)\beta_s l}{\gamma_s^2 c}+\frac{W_6I'_l(\gamma_s c)\omega\mu_0}{\gamma_s}+\frac{W_4K_l(\gamma_s c)\beta_s l}{\gamma_s^2 c}+\frac{W_8K'_l(\gamma_s c)\omega\mu_0}{\gamma_s},$$
$$W_1=S_{13}R_5+S_{14}R_7+S_{15}R_2+S_{16}R_4,$$
$$W_2=S_{13}R_6+S_{14}R_8+S_{15}R_1+S_{16}R_3,$$
$$W_3=S_9R_5+S_{10}R_7+S_{11}R_2+S_{12}R_4,$$

$$W_4 = S_9 R_6 + S_{10} R_8 + S_{11} R_1 + S_{12} R_3,$$
$$W_5 = S_5 R_2 + S_6 R_4 + S_7 R_5 + S_8 R_7,$$
$$W_6 = S_5 R_1 + S_6 R_3 + S_7 R_6 + S_8 R_8,$$
$$W_7 = S_1 R_2 + S_2 R_4 + S_3 R_5 + S_4 R_7,$$
$$W_8 = S_1 R_1 + S_2 R_3 + S_3 R_6 + S_4 R_8,$$
$$R_1 = \frac{\gamma'_s}{\omega\varepsilon}\left(\frac{\beta_s l}{\gamma_s^2 a} - \frac{\beta_s l}{\gamma_s^2 a}\right)\frac{I_l(\gamma_s a)K_l(\gamma'_s a)}{P(\gamma'_s a)},$$
$$R_2 = \frac{I_l(\gamma_s a)K'_l(\gamma'_s a)}{P(\gamma'_s a)} - \frac{\varepsilon_0\gamma'_s}{\varepsilon\gamma_s}\frac{I'_l(\gamma_s a)K_l(\gamma'_s a)}{P(\gamma'_s a)},$$
$$R_3 = \frac{\gamma'_s}{\omega\varepsilon}\left(\frac{\beta_s l}{\gamma_s^2 a} - \frac{\beta_s l}{\gamma'^2_s a}\right)\frac{I_l(\gamma_s a)I_l(\gamma'_s a)}{P(\gamma'_s a)},$$
$$R_4 = -\frac{I_l(\gamma_s a)I'_l(\gamma'_s a)}{P(\gamma'_s a)} + \frac{\varepsilon_0\gamma'_s}{\varepsilon\gamma_s}\frac{I'_l(\gamma_s a)I_l(\gamma'_s a)}{P(\gamma'_s a)},$$
$$R_5 = \frac{\gamma'_s}{\omega\mu_0}\left(\frac{\beta_s l}{\gamma'^2_s a} - \frac{\beta_s l}{\gamma_s^2 a}\right)\frac{I_l(\gamma_s a)K_l(\gamma'_s a)}{P(\gamma'_s a)},$$
$$R_6 = \frac{I_l(\gamma_s a)K'_l(\gamma'_s a)}{P(\gamma'_s a)} - \frac{\gamma'_s}{\gamma_s}\frac{I'_l(\gamma_s a)K_l(\gamma'_s a)}{P(\gamma'_s a)},$$
$$R_7 = \frac{\gamma'_s}{\omega\mu_0}\left(\frac{\beta_s l}{\gamma_s^2 a} - \frac{\beta_s l}{\gamma'^2_s a}\right)\frac{I_l(\gamma_s a)I_l(\gamma'_s a)}{P(\gamma'_s a)},$$
$$R_8 = -\frac{I_l(\gamma_s a)I'_l(\gamma'_s a)}{P(\gamma'_s a)} + \frac{\gamma'_s}{\gamma_s}\frac{I'_l(\gamma_s a)I_l(\gamma'_s a)}{P(\gamma'_s a)},$$
$$P(\gamma'_s a) = K'_l(\gamma'_s a)I_l(\gamma'_s a) - K_l(\gamma'_s a)I'_l(\gamma'_s a),$$
$$S_1 = \frac{\gamma_s}{\omega\mu_0}\left(\frac{\beta_s l}{\gamma'^2_s b} - \frac{\beta_s l}{\gamma_s^2 b}\right)\frac{I_l(\gamma'_s b)I_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_2 = \frac{\gamma_s}{\omega\mu_0}\left(\frac{\beta_s l}{\gamma'^2_s b} - \frac{\beta_s l}{\gamma_s^2 b}\right)\frac{K_l(\gamma'_s b)I_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_3 = \frac{\gamma_s}{\gamma'_s}\frac{I'_l(\gamma'_s b)I_l(\gamma_s b)}{Q(\gamma_s b)} - \frac{I_l(\gamma'_s b)I'_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_4 = \frac{\gamma_s}{\gamma'_s}\frac{K'_l(\gamma'_s b)I_l(\gamma_s b)}{Q(\gamma_s b)} - \frac{K_l(\gamma'_s b)I'_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_5 = \frac{\gamma_s}{\omega\mu_0}\left(\frac{\beta_s l}{\gamma_s^2 b} - \frac{\beta_s l}{\gamma'^2_s b}\right)\frac{I_l(\gamma'_s b)K_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_6 = \frac{\gamma_s}{\omega\mu_0}\left(\frac{\beta_s l}{\gamma_s^2 b} - \frac{\beta_s l}{\gamma'^2_s b}\right)\frac{K_l(\gamma'_s b)K_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_7 = \frac{I_l(\gamma'_s b)K'_l(\gamma_s b)}{Q(\gamma_s b)} - \frac{\gamma_s}{\gamma'_s}\frac{I'_l(\gamma'_s b)K'_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_8 = \frac{K_l(\gamma'_s b)K'_l(\gamma_s b)}{Q(\gamma_s b)} - \frac{\gamma_s}{\gamma'_s}\frac{K'_l(\gamma'_s b)K_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_9 = \frac{\gamma_s}{\omega\varepsilon_0}\left(\frac{\beta_s l}{\gamma'^2_s b} - \frac{\beta_s l}{\gamma_s^2 b}\right)\frac{I_l(\gamma'_s b)I_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_{10} = \frac{\gamma_s}{\omega\varepsilon_0}\left(\frac{\beta_s l}{\gamma'^2_s b} - \frac{\beta_s l}{\gamma_s^2 b}\right)\frac{K_l(\gamma'_s b)I_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_{11} = \frac{\varepsilon}{\varepsilon_0}\frac{\gamma_s}{\omega\gamma'_s}\frac{I'_l(\gamma'_s b)I_l(\gamma_s b)}{Q(\gamma_s b)} - \frac{I_l(\gamma'_s b)I'_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_{12} = \frac{\varepsilon}{\varepsilon_0}\frac{\gamma_s}{\omega\gamma'_{s0}}\frac{K'_l(\gamma'_s b)I_l(\gamma_s b)}{Q(\gamma_s b)} - \frac{K_l(\gamma_s b)I'_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_{13} = \frac{\gamma_s}{\omega\varepsilon_0}\left(\frac{\beta_s l}{\gamma_s^2 b} - \frac{\beta_s l}{\gamma'^2_s b}\right)\frac{I_l(\gamma'_s b)K_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_{14} = \frac{\gamma_s}{\omega\varepsilon_0}\left(\frac{\beta_s l}{\gamma_s^2 b} - \frac{\beta_s l}{\gamma'^2_s b}\right)\frac{K_l(\gamma'_s b)K_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_{15} = \frac{I_l(\gamma'_s b)K'_l(\gamma_s b)}{Q(\gamma_s b)} - \frac{\varepsilon}{\varepsilon_0}\frac{\gamma_s}{\gamma'_s}\frac{I'_l(\gamma'_s b)K_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$S_{16} = \frac{K_l(\gamma'_s b)K'_l(\gamma_s b)}{Q(\gamma_s b)} - \frac{\varepsilon}{\varepsilon_0}\frac{\gamma_s}{\gamma'_s}\frac{K'_l(\gamma'_s b)K_l(\gamma_s b)}{Q(\gamma_s b)},$$
$$Q(\gamma_s b) = K'_l(\gamma_s b)I_l(\gamma_s b) - K_l(\gamma_s b)I'_l(\gamma_s b)$$

同样,4 区场也可用切向电场分量在环面上($r = c$)的幅值 A_{ls}、C_{ls}表示为

$$E_{z4}(r,\theta,z) = \sum_{l=1,3,5,}^{+\infty}\sum_{s=-\infty}^{+\infty} A_{ls}\frac{X_l(r,r_s)}{X_l(a,r_s)}\cos(l\theta)e^{-j\beta_s z}, \tag{11}$$

$$E_{z4}(r,\theta,z) = \sum_{l=1,3,5,.}^{+\infty}\sum_{s=-\infty}^{+\infty} -\frac{\gamma_s}{\omega\mu_0}\left[C_{ls} + \frac{\beta_s l}{\gamma_s^2 c}A_{ls}\right]\frac{Y_s(r,r_s)}{Y'_l(c,r_s)}\sin(l\theta)e^{-j\beta_s z}, \tag{12}$$

其中

$$X_l(r,r_s) = I_l(\gamma_s r)K_l(\gamma_s r_s) - I_l(\gamma_s r_s)K_l(\gamma_s r),$$
$$Y_l(r,r_s) = I_l(\gamma_s r)K'_l(\gamma_s r_s) - I'_l(\gamma_s r_s)K_l(\gamma_s r).$$

2　考虑空间电荷效应的热色散方程

由于环板慢波结构的边界很复杂,采用变分法的特殊方法——Ritz-Galerkin 法,求解边值问题,从而获得热色散方程.

选取环间隙内($r = c$, $|z| \leqslant g$)的基函数为

$$\begin{cases} e_{z(q'n')} = \begin{cases} \cos q'\theta\cos\dfrac{n'\pi z}{g}, & |\theta| \leqslant \pi/2 \\ -g \leqslant z \leqslant g\ (q' = 1,3,5\ldots, & n' = 0,1,2\ldots) \\ -\cos q'\theta\cos\dfrac{n'\pi z}{g}, & \pi/2 \leqslant |\theta| \leqslant \pi \\ 0 & g \leqslant z \leqslant p - g \end{cases} \\ e_{\theta(q'n')} = \begin{cases} \sin q'\theta\sin\dfrac{n'\pi z}{g}, & |\theta| \leqslant \pi/2 \\ -g \leqslant z \leqslant g\ (q' = 1,3,5\ldots, & n' = 1,2,3\ldots) \\ -\sin q'\theta\sin\dfrac{n'\pi z}{g}, & \pi/2 \leqslant |\theta| \leqslant \pi \\ 0 & g \leqslant z \leqslant p - g \end{cases} \end{cases} \tag{13}$$

在环面($r = c$)上,3 区的切向电场可以表示为

所有基函数的线性迭加

$$\begin{cases} E_{z3}(c) = \sum\limits_{q=1,3,5}^{+\infty} \sum\limits_{n'=0,1,2}^{+\infty} u_{q'n'} e_{z(q'n')}, \\ E_{\theta 3}(c) = \sum\limits_{q=1,3,5}^{+\infty} \sum\limits_{n'=1,2,3}^{+\infty} v_{q'n'} e_{\theta(q'n')}, \end{cases} \tag{14}$$

其中 $u_{q'n}$、$v_{q'n'}$ 为展开系数.

在任一周期中,具有相同标号(q'、n')的基函数可以展开为下列空间谐波的迭加

$$\begin{cases} e_{z(q'n')} = \sum\limits_{l=0,2,4}^{+\infty} \sum\limits_{s=-\infty}^{+\infty} \Phi_l(q')\Phi_s(n')\cos l\theta e^{-j\beta_s z}, \\ e_{\theta(q'n')} = \sum\limits_{l=2,4,6}^{+\infty} \sum\limits_{s=-\infty}^{+\infty} \Psi_l(q')\Psi_s(n')\sin l\theta e^{-j\beta_s z}, \end{cases} \tag{15}$$

$\Phi_1(q')$, $\Phi_s(n')$, $\Psi_1(q')$, $\Psi_s(n')$ 为傅里叶展开系数

$$\begin{cases} \Phi_l(q') = \dfrac{4q'(-1)^{(l+q+1)/2}}{\pi(1+\delta_l)(l^2-q'^2)}, \\ \Psi_l(q') = \dfrac{4l(-1)^{(l+q+1)/2}}{\pi(l^2-q'^2)}, \\ \Phi_s(n') = \dfrac{2g}{p}\,\dfrac{(-1)^n}{1-(n'\pi/\beta_s g)^2}\,\dfrac{\sin\beta_s g}{\beta_s g}, \\ \Psi_s(n') = j\,\dfrac{2g}{p}\,\dfrac{(-1)^n}{1-(n'\pi/\beta_s g)^2}\,\dfrac{n'\pi}{\beta_s g}\,\dfrac{\sin\beta_s g}{\beta_s g}, \end{cases} \tag{16}$$

其中

$$\delta_l = \begin{cases} 1 & l=1 \\ 0 & l\neq 1 \end{cases}$$

利用式(9)、式(1)及式(13)~式(16)诸式,经过复杂的推导,可得

$$\begin{cases} F_{ls} = \sum\limits_{q'=1,3,5}^{+\infty} \sum\limits_{n=0,1,2}^{+\infty} \Phi_l(q')\Phi_s(n')u_{q'n}, \\ H_{ls} = -j\sum\limits_{q'=1,3,5}^{+\infty} \sum\limits_{n=1,2,3}^{+\infty} \Psi_l(q')\Psi_s(n')v_{q'n'}, \end{cases} \tag{17}$$

这里 l 为偶数.

利用相同的方法,同样可以得到

$$\begin{cases} A_{ls} = \sum\limits_{q=0,1,2}^{+\infty} \delta_{lq}\Phi_s(n')u_{q'n}, \\ C_{ls} = -j\sum\limits_{q'=1,2,3}^{+\infty} \delta_{lq}\Psi_s(n')v_{q'n'}, \end{cases} \tag{18}$$

在环间隙($r=c$, $|z|\leqslant g$ 内,3 区和 4 区的切向磁场满足如下的边界条件

$$\begin{cases} H_{\theta 3}\big|_{r=c} - H_{\theta 4}\big|_{r=c} = 0, \\ H_{Z3}\big|_{r=c} - H_{Z4}\big|_{r=c} = 0, \end{cases} \tag{19}$$

分别用 $e_{z(q'n')}$、$e_{\theta(q'n')}$ 作权函数,对式(19)中两式作内积,可得一关于 $u_{q'n'}$, $v_{q'n'}$ 的无限维齐次线性方程组

$$\begin{vmatrix} [A^{11}_{ij}][A^{12}_{ij}] \\ [A^{21}_{ij}][A^{22}_{ij}] \end{vmatrix} \begin{vmatrix} [U] \\ [V] \end{vmatrix} = 0, \tag{20}$$

其中 $[U]$、$[V]$ 是关于 $u_{q'n}$, $v_{q'n'}$ 的列向量; $[A^{11}_{ij}]$、$[A^{12}_{ij}]$、$[A^{21}_{ij}]$、$[A^{22}_{ij}]$ 为四个块矩阵. 每一组(i,j)分别与不同的(q,n)及(q',n')相对应,块矩阵元素可分别表示为

$$A^{11}_{ij} = (-1)^{n+n'}\sum_{s=-\infty}^{+\infty}\frac{(\sin\beta_s g/\beta_s g)^2}{[1-(n\pi/\beta_s g)^2][1-(n'\pi/\beta_s g)^2]}\cdot \left\{\frac{k\delta_{qq'}}{\gamma_s\eta_0}\left[\frac{X'_q(c,r_s)}{X_q(c,r_s)} - \frac{Y_q(c,r_s)}{Y_{q'}(c,r_s)}\left(\frac{\beta_s c q}{\gamma_s c k c}\right)^2\right] + (-1)^{(q+q')/2}\frac{16qq'}{\pi^2}\cdot \sum_{l=0,2,4}^{+\infty}\frac{M_1+M_3}{(1+\delta_l)^2(l^2-q^2)(l^2-q'^2)}\right\}, \tag{21}$$

$$A^{12}_{ij} = (-1)^{n+n'+1}\sum_{s=-\infty}^{+\infty}\frac{(\sin\beta_s g/\beta_s g)^2(n'\pi/\beta_s g)}{[1-(n\pi/\beta_s g)^2][1-(n'\pi/\beta_s g)^2]}\cdot \left\{\frac{\delta_{qq'}\beta_s q}{\gamma_s c k\eta_0}\,\frac{Y_q(c,r_s)}{Y'_q(c,r_s)} - (-1)^{\frac{q+q'}{2}}\cdot \sum_{l=2,4,6}^{+\infty}\frac{16ql(M_2+M_4)}{\pi^2(l^2-q^2)(l^2-q'^2)}\right\}, \tag{22}$$

$$A^{21}_{ij} = (-1)^{n+n'}\sum_{s=-\infty}^{+\infty}\frac{(\sin\beta_s g/\beta_s g)^2(n\pi/\beta_s g)}{[1-(n\pi/\beta_s g)^2][1-(n'\pi/\beta_s g)^2]}\cdot$$

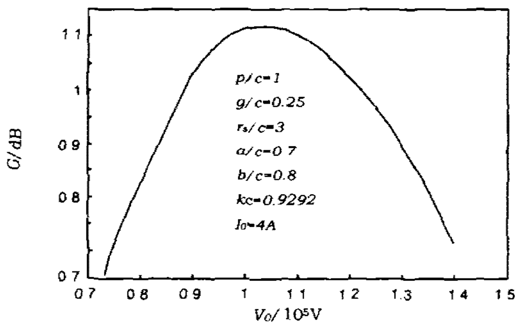

图 2 电子注电压对小信号增益的影响
Fig.2 The influence of the electron beam voltage on the small signal gain

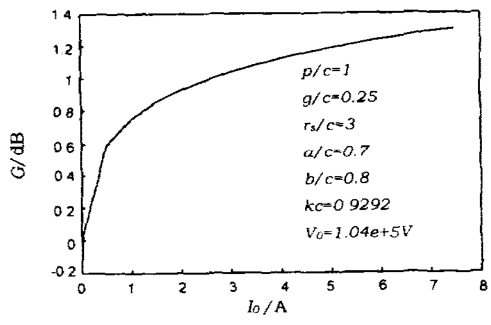

图 3 电子注电流对小信号增益的影响
Fig.3 The influence of the electron beam current on the small signal gain

$$\left\{\frac{\delta_{qq'}\beta_s q}{\gamma_s c k\eta_0}\frac{Y_q(c,r_s)}{Y'_q(c,r_s)}-(-1)^{\frac{q+q'}{2}}\sum_{l=2,4,6}^{+\infty}\frac{16\gamma_s^2 c q' l M_1}{\pi^2\beta_s l(l^2-q^2)(l^2-q'^2)}\right\}, \tag{23}$$

$$A_{ij}^{22}=(-1)^{n+n'}\sum_{s=-\infty}^{+\infty}\frac{(\sin\beta_s g/\beta_s g)^2 nn'\pi^2}{[1-(n\pi/\beta_s g)^2][1-(n'\pi/\beta_s g)^2](\beta_s g)^2}\cdot$$
$$\left\{\frac{\gamma_s\delta_{qq'}}{k\eta_0}\frac{Y_q(c,r_s)}{Y'_q(c,r_s)}-(-1)^{\frac{q+q'}{2}}\sum_{l=2,4,6}^{+\infty}\frac{16\gamma_s^2 c l^2 M_2}{\pi^2\beta_s l(l^2-q^2)(l^2-q'^2)}\right\}. \tag{24}$$

其中

$$\delta_{qq'}=\begin{cases}1 & q=q'\\ 0 & q\neq q'\end{cases}$$

$$M_1=\frac{\beta_s l}{\gamma_s^2 c}\left[\frac{Q_2(W_5 I_l(\gamma_s c)+W_7 K_l(\gamma_s c))}{P_1Q_2-P_2Q_1}+\frac{Q_1(W_6 I_l(\gamma_s c)+W_8 K_l(\gamma_s c))}{P_2Q_1-P_1Q_2}\right],$$

$$M_2=\frac{\beta_s l}{\gamma_s^2 c}\left[\frac{P_2(W_5 I_l(\gamma_s c)+W_7 K_l(\gamma_s c))}{P_1Q_2-P_2Q_1}+\frac{P_1(W_6 I_l(\gamma_s c)+W_8 K_l(\gamma_s c))}{P_2Q_1-P_1Q_2}\right],$$

$$M_3=\frac{\omega\varepsilon_0}{\gamma_s}\left[\frac{Q_2(W_1 I'_l(\gamma_s c)+W_3 K'_l(\gamma_s c))}{P_1Q_2-P_2Q_1}+\frac{Q_1(W_2 I'_l(\gamma_s c)+W_4 K'_l(\gamma_s c))}{P_2Q_1-P_1Q_2}\right],$$

$$M_4=\frac{\omega\varepsilon_0}{\gamma_s}\left[\frac{P_2(W_1 I'_l(\gamma_s c)+W_3 K'_l(\gamma_s c))}{P_1Q_2-P_2Q_1}+\frac{P_1(W_2 I'_l(\gamma_s c)+W_4 K'_l(\gamma_s c))}{P_2Q_1-P_1Q_2}\right],$$

方程式(20)中的[U]、[V]有非零解的条件是

$$\det\begin{vmatrix}[A_{ij}^{11}] & [A_{ij}^{12}]\\ [A_{ij}^{21}] & [A_{ij}^{22}]\end{vmatrix}=0. \tag{25}$$

这就是考虑了电子束空间电荷效应的环板行波管的热色散方程,数值求解上述热色散方程可得小信号增益与电子注参量及环板结构几何参数之间的关系.

3　数值模拟结果与讨论

热色散方程式(25)中的矩阵块元素是 M_1、M_2、M_3、M_4 的函数,而由 M_1、M_2、M_3、M_4 的表达式知,它们都是电子束区域的径向传播常数 γ'_s 的函数.我们知道

$$\gamma'^2_s=\gamma_s^2\left[1-\frac{\omega_p^2}{(\omega-\beta_s v_0)^2}\right], \tag{26}$$

考虑相对论效应,则有

$$\begin{cases}\omega_p^2=\dfrac{e\rho_0}{\gamma^3 m\varepsilon_0},\\ \dfrac{1}{2}\gamma m v_0^2=eV_0.\end{cases} \tag{27}$$

其中 $\gamma=\dfrac{1}{\sqrt{1-(v_0/v_c)^2}}$ 为相对论因子,$\rho_0=\dfrac{I_0}{v_0\pi(b^2-a^2)}$为束电荷密度,$v_c$ 是光速.

整理式(26)和式(27)得

$$\begin{cases}\gamma'^2_s=(\beta_s^2-k^2)\left\{1-\dfrac{\eta I_0}{\gamma^3\varepsilon_0 v_0\pi[(b/c)^2-(a/c)^2][kc\cdot v_c-\beta_s c\cdot v_0]^2}\right\},\\ v_0=\dfrac{\sqrt{2\eta V_0(\sqrt{\eta^2V_0^2+v_c^4}-\eta V_0)}}{v_c}.\end{cases} \tag{28}$$

由式(28)可知,热色散方程是包含电子注参量 I_0、V_0、a、b 与环板结构几何参数的复数超越方程.我们可以给定这些参量,解出基波复数传播常数的复数解 $\beta_0 c$,由此可求得环板行波管每个周期的增益为

$$G=20\log[e^{Im(\beta_0 c)(p/c)}]=8.686\,\frac{q}{c}\mathrm{Im}(\beta_0 c). \tag{29}$$

数值计算中,只需少数几项基函数就可以得到满意的精度.实际计算时,我们取 3 项,结果如图 2

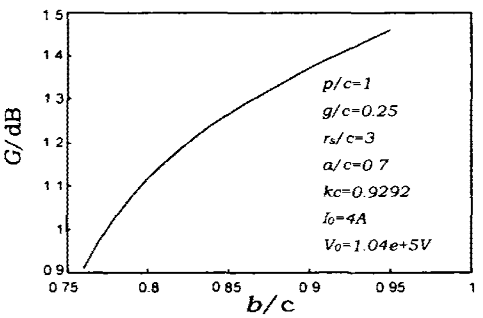

图 4　电子注外径对小信号增益的影响
Fig.4 The influence of the outer radius of electron beam on the small signal gain

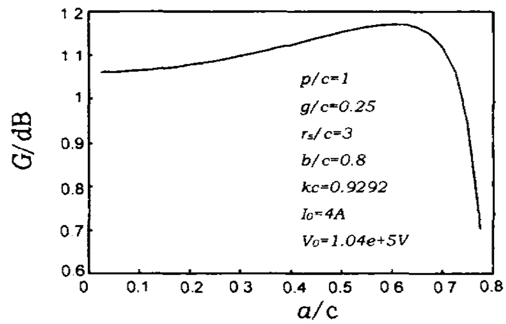

图 5　电子注内径对小信号增益的影响
Fig.5 The influence of the inner radius of electron beam on the small signal gain

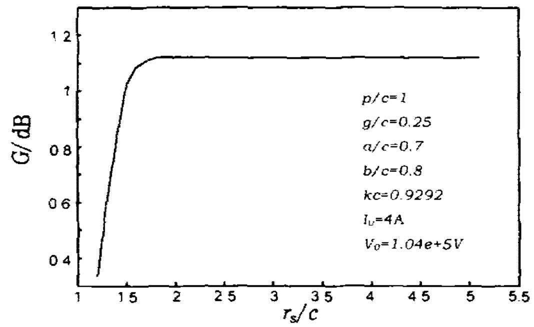

图 6 屏蔽筒半径对小信号增益的影响
Fig.6 The influence of the metal envelope radius on the small signal gain

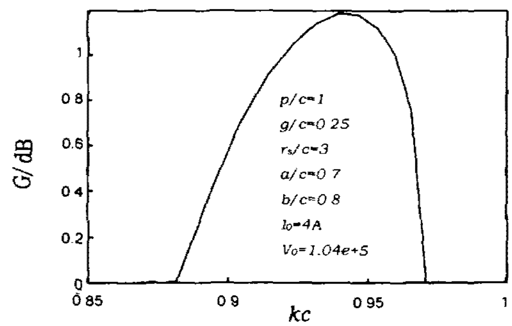

图 7 小信号增益随频率的变化
Fig.7 The relation between the small signal gain and the normalized frequency

~图 7 所示.

通过以上的数值模拟,我们得到以下结论:

(1) 从图 2 可知,对应最大增益,有一饱和注电压存在,饱和电压达 104 kV,这与文献[4]的试验结果基本一致,证明了本文理论分析的正确性.

(2) 从图 3 可知,小信号增益随电子注电流增加而增加.这是因为电流越大,则皮尔斯增益参量越大,因而增益越大.

(3) 由图 4 可知,电子注外半径对小信号增益影响强烈.随着电子注外半径的增大,注表面的纵向电场随之增大,注波互作用增强,增益也就增大.

(4) 从图 5 可知,其它参量不变,小信号增益随电子注内半径增大而增大.这是因为注电流不变,内径增大,电子密度增大,与注表面较强纵向电场作用的电子增多,导致注波互作用增强,增益增大,当束内径增大到接近束外径时,计算所得增益剧烈下降,这从一个方面证明了采用无限薄电子束与有厚度电子束的计算结果的差异,采用无限薄电子束计算的增益将远小于器件的实际增益.这主要是因为若不考虑束的厚度,即采用无限薄电子束模型,束的空间电荷效应与实际空间电荷效应差异很大,导致所计算的增益与实际增益相差较大.

(5) 从图 6 可知,小信号增益随屏蔽筒半径的增大而增大,当屏蔽筒半径大于某一值时,对小信号增益影响极弱.

(6) 图 7 是增益与频率的关系曲线,可知,环板行波管具有增益大,频带窄的特点,更适合用作大功率毫米波行波管;另外,通过该图粗略的估计行波管的瞬时带宽,合理选择电子注参量和慢波系统的几何尺寸,可以获得相对宽的带宽.

4 结语

本文运用场匹配和变分原理相结合的方法,研究了相对论空心电子束环板行波管的小信号特性,给出了其热色散方程,并进行了数值模拟,讨论了环板慢波系统几何尺寸和电子束参量对小信号增益的影响.研究表明:对应最大增益,有一饱和注电压存在;小信号增益随电子注电流增加而增加;电子注半径对小信号增益影响强烈,随电子注外半径的增大,小信号增益也增大;环板行波管具有高增益、窄频带的特点,适合用作大功率毫米波行波管;合理的选择电子注参量和慢波系统的几何尺寸,可以使环板行波管既获得大的功率输出,又获得相对较宽的带宽.同时指出采用无限薄电子束计算的增益与器件的实际增益相比,具有很大的偏差.

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