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CONTOURS EMBELLISHMENT USING ADAPTIVE CUBIC B-SPLINE IN IMAGE SEGMENTATION

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Abstract A novel method to get smooth contour in image segmentation process was presented. First, dynamic programming algorithm was adopted to process the least cumulative cost matrix of object image, and the optimum contour was extracted along the direction that the cumulative costs descend fastest. Then the contour was smoothened and fitted by using adaptive cubic B-spline, which adjusts the distribution of control points adaptively according to the contour curvature. Experiment results of many images showed that this method could effectively suppress tiny zigzags on contours and could produce smoother contour curve than other previous methods without losing the fine structures of the contours at the same time.

Key words contour extraction, image segmentation, B-spline, adaptive.

图像分割中采用自适应 3 次 B 样条修饰轮廓线

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摘要 提出了一种在图像分割中获取连续光滑轮廓线的方法. 该方法先以动态规划方法处理图像的全局最小累积代价阵，并从累积代价梯度降低最快的方向提取全局最优的轮廓线. 然后用一种自适应 3 次 B 样条对获得的轮廓线进行修饰和平滑处理. 该样条可根据轮廓线不同处的曲率变化情况，自适应地调整控制点的分布. 在各类图像上的试验表明，该方法能有效地消除轮廓线上的小锯齿，获得较其它方法更平滑的曲线，同时又保留了轮廓线的特征细节.

关键词 轮廓提取，图像分割，B 样条，自适应.

Introduction

The contour is a fundamental feature of an image. It is termed as a set of the pixels, which contrast with the near pixels apparently in gray values. The contours consist of the regions that exist between different objects, different areas, or different basic units. They are the most important features which image segmentation depends on, and they are also the important information source of texture feature and the basis of shape feature analysis.

Most previous algorithms of contour detection obtain objects according to the pixel gray value or gray-related parameters. In many cases, the detected object contours are far from smoothness and with many tiny abnormal zigzags due to the noise, quantization error or the gradient distribution of gray scale in images. It does not accord with the real situations of the object, and it will result in a lot of inconveniences in subsequent processings. To get over these problems, it is essential to adopt some smoothing or embellishing method on the obtained contours to make the curves approaching realities. If the smoother contours can be obtained by importing small-correcting data, the obtained

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results are acceptable in many cases. And these will bring advantages to the subsequent processings, such as the image recognition, computer vision, remote sensing and 3-D reconstruction, etc..

Aiming at the previous discussion, we present a subdivision algorithm for contour embellishment by using adaptive cubic B-spline. First, we adopt a dynamic programming (DP) algorithm to process the object image in order to get an optimum contour, then smooth and embellish the contour with adaptive B-spline, which can adjust the distribution of control points according to the contour curvature by importing small-correcting data. Experiments of many images demonstrate that the method can represent the image contours accurately.

1 Cubic B-spline in image segmentation

Here, we adopt C^2 continual cubic B-spline in contour fitting, which changes smoothly and can describe the image contour feature well in curve analysis. For a group of control points (C_0, C_1, ... C_{N-1}), the closed B-spline can be described as[1,2]

$$P(s) = \sum_{n=0}^{N=2} C_n B_{n,4}(s),$$
$$C_{-1} = C_{N-1},\ C_N = C_0,\ s \in [0,N] \qquad (1)$$

where $B_{n,4}(s)$ is the basic function of cubic B-spline, s is a real number between 0 and N. B-spline is local, that is, for a cubic B-spline, 4 knots can determine a line section (Fig. 1). The terminal points are the control point will only affect a line section controlled by this point. The relationship between the knots and the control points is

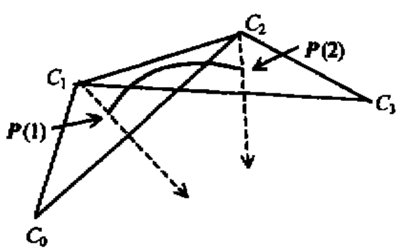

Fig. 1 The cubic B-spline curve
图1 3次B样条曲线

$$P(i) = C_{i-1}/6 - 2C_i/3 + C_{i+1}/6 \qquad (2)$$

$$P'(i) = (C_{i-1} - C_{i-1})/2,$$
$$P''(i) = (C_{i+1} - C_i) + (C_{i-1} - C_i). \qquad (3)$$

when the control points are known, every 4 control points can determine a line section. The cubic B-spline matrix expression is

$$P(s-i) = \frac{1}{6}[s^3 \quad s^2 \quad s \quad 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} C_{i-1} \\ C_i \\ C_{i-1} \\ C_{i-2} \end{bmatrix}.$$
$$0 \leqslant s \leqslant 1 \qquad (4)$$

when the closed curve knots are known, a matrix can be constructed as follows.

$$\begin{bmatrix} 4 & 1 & 0 & \cdots & -1 \\ -1 & 4 & 1 & \cdots & 0 \\ & \vdots & & \vdots & \\ 1 & 0 & \cdots & -1 & 4 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ \vdots \\ C_{N-1} \end{bmatrix} = \begin{bmatrix} 6P(0) \\ 6P(1) \\ \vdots \\ 6P(N-1) \end{bmatrix}. \qquad (5)$$

By solving the above linear equations, the control points and the corresponding curve can be obtained. It needs enough control points to describe the real contour. Because of the continuity and the slickness, B-spline cannot describe the curve near corners well with limited uniform control points. At some places where the curvature of curve is big, such as the sections near corner or angle, two or three coincidence control points are adopted to approximate these line sections. On the contrary, at the gentle sections, we can use only a few control points to fit them. For these non-uniform distributed control points, a criterian which can adjust the distribution of these points adaptively is necessary.

2 Adaptive adjustment of the control knots of B-spline

When B-spline is adopted to fit contours, the discrete points that take part in the iterative calculation correspond to the knots in B-spline, and the control points can be obtained through the equation (5). To improve the algorithm's efficiency, we take the equal-interval sampling discrete points as the initial control points C^0, which determine the knots of B-spline as the initial discrete vectors V^0. From the time $t=0$, the iteration process is as follows:

Step 1: The control points C^t are sampled in equal interval on the obtained contour curve.

Step 2: B-spline knots are determined by these control points, which are taken as a group of discrete vectors V^t.

Step 3: For the uniform distributed control points, a uniform B-spline $p(s)$ is obtained. There are M points $q(i)$, $i=0\cdots M-1$, on the discrete curve. We assume that the distribution function and the local cumulative function of every point at the uniform curve are $f_q(i)=1/M$ and $F_q(i)=\sum_{i=0}^{i} f_q(i)$. The redistribution of curve control points is adjusted depending on the curve curvature. The points are distributed more at the places with big curvature, and less at the places with small curvature. The distribution function of control points is directly proportional to the curvature. The function is defined as

$$f_{q'}(i)=k(i)/\sum_{i=0}^{M-1}k(i), \tag{6}$$

where $q'(i)$ is the redistributed point, and the curvature $k(i)$ is defined as

$$k(i)=(\Delta\theta(i))^2 * G_\sigma(i), \tag{7}$$

$$\Delta\theta(i)=\cos^{-1}\frac{\vec{x}_i\vec{x}_{i-1}}{|\vec{x}_i|\cdot|\vec{x}_{i-1}|}=$$

$$\cos^{-1}\frac{(x_i-x_{i-1})(x_{i+1}-x_i)+(y_i-y_{i-1})(y_{i+1}-y_i)}{\sqrt{[(x_i-x_{i-1})^2+(y_i-y_{i-1})^2][(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2]}}. \tag{8}$$

where $G_\sigma(i)$ is Guassian function, which makes the variation of curvature gentle. And the cumulative distribution function will not change suddenly at the places where the curvature is big. The reason why the curvature formula doesn't be expressed as a traditional differential form is that higher derivative for discrete points calculation may import errors, which make the curvature change acutely at some places with big curvature. But the above equation changes the curvature gently. The cumulative distribution function $F_{q'}(i)$ can be constructed by using equation (6). Then the sampling transform function of non-uniform control points for the curvature adjustment is

$$T(i)=F_{q'}^{-1}(i), \quad i\in[0, M-1] \tag{9}$$

Supposing sampling interval is t when the interval is equal, the control point $C_i=q(t*i)$, $i\in[0, N-1]$, can be obtained. According to the obtained non-uniform sampling transform function, a group of non-uniform control points can be gained:

$$C'_i=q(T(t*i)). \quad i\in[0, N-1] \tag{10}$$

Step 4: Adopting the changed control points C^{t+1} to obtain a new contour. Repeat this process until the contour curve is stable.

Through the redistribution, there are enough control points to fit the real contour at the places where the curvature is big. But at the places with small curvature, the control points become sparse. So, the distribution of B-spline control points is adaptively adjusted by the distribution of curvature.

3 Contour detection by DP method

There are many kinds of methods for detecting the object image contour. To minimize the output uncertainty of a medical image analysis system, taking the optimum criterions into account is necessary. Dynamical programming (DP) is a powerful tool in multistage decision. Barrett and Udupa *et al.* in Refs. [3]~[6] firstly used DP method in contour detection. Their results confirmed that the optimum method is valid in contour detection and tracking. This paper adopts the DP method, which can obtain the global optimum contour. This method is robust for noisy images and can get continual and closed contour.

The DP algorithm in contour detection uses the cumulative cost *cum* (x_N, y_N) between the starting point $P_s(x_0, y_0)$ and the end point $P_e(x_N, y_N)$ as the objective function, and the nodes of the initial cost matrix $C[M, N]$ as the variables. The optimum value of these variables makes the cumulative cost of end point minimum. The model is a kind of indeterminacy multistage decision procession.

DP algorithm utilizes the information of local gradient and global contour cumulating cost to detect the object contour. This is the reason why the algorithm can obtain the global optimum solutions[5,6]. The algorithm includes the following three steps:

Step 1: Calculating the initial cost matrix. From the original image gray value matrix, which is operated by the fuzzy threshold filter and the gradient operator, the initial cost matrix $C[M, N]$ can be obtained.

Step 2: Calculating the minimal cumulative cost matrix. According to the DP model, user specified the starting point P_s near the edge. Every node in the matrix is initialized to a big value except P_s, which is initialized to 0. k denotes the iteration times. Every point is checked in each time of iteration calculation. If there is any updated value in the eight neighbors of a point, use the least one to calculate the cost value of the center point by the equation

$$cum_{k+1}(x_{k+1}, y_{k+1}) = \min_{(x_k, y_k) \in N(x_{k-1}, y_{k-1})} [C(x_{k+1}, y_{k-1}) + cum_k(x_k, y_k)], \tag{11}$$

where $cum(x_k, y_k)$ is the cumulated cost of the center point, $N(x_{k-1}, y_{k+1})$ is the neighbor set of the point (x_{k+1}, y_{k-1}), $C(x_{k-1}, y_{k+1})$ is a point in the $C[M, N]$, and $k+1$ indicates the iteration number. This process is repeated for every iteration until the system converges. At the end of the step, we obtain the minimum cumulative matrix. The starting point has the global minimal energy, and there is a path in which the cost gradient descends fastest among all possible paths between the point P_s and P_e. It is the optimum path and desired connected contour between P_s and P_e.

Step 3: Trace back along the minimum cost value gradient. From a specified end point P_e, the starting point P_s is reached, along the path that the minimum cumulative cost descends fastest. The desired global optimum contour between P_s and P_e is obtained.

The path of cumulative cost minimization obtained by DP method is illustrated in Fig. 2.

4 Results and discussion

Figure 3 shows the experiment results obtained by the presented method. It illustrates that adopting B-spline to embellish obtained image contours can smoothen the curve and keep its feature to the largest extension. This is very important in computer vision and images comprehending. Especially for those images with noises and dim edges, the contours obtained by using many other methods are far from smoothness. The situation can be improved obviously by the present B-spline

162	94	90	87	101	111	101	120	113
202	107	87	88	96	101	99	115	115
228	160	98	84	95	94	104	122	121
241	215	142	87	78	73	94	113	103
240	228	210	125	84	77	84	95	95
244	237	226	188	90	81	81	94	86
241	234	236	234	183	103	77	77	82
234	231	236	239	215	175	115	78	73
240	228	231	236	228	222	198	142	98
239	228	230	236	231	237	224	217	198

(a)

4	8	21	21	21	21	21	17	21
7	3	21	21	21	21	21	16	21
13	2	8	21	21	21	21	16	21
21	7	3	10	21	15	14	15	17
21	15	6	2	12	21	16	15	17
21	21	15	4	4	21	21	21	21
21	21	21	12	1	4	16	21	21
21	21	21	21	10	3	5	13	21
21	21	21	21	16	11	4	2	8
21	21	21	21	21	21	13	6	2

(b)

4	8	21	42	*	*	*	*	*
7	0	21	29	*	*	*	*	*
13	2	8	26	*	*	*	*	*
23	9	5	15	*	*	*	*	*
*	*	*	*	*	*	*	*	*
*	*	*	*	*	*	*	*	*
*	*	*	*	*	*	*	*	*
*	*	*	*	*	*	*	*	*
*	*	*	*	*	*	*	*	*
*	*	*	*	*	*	*	*	*

(c)

4	8	21	42	50	68	78	87	92
7	0	21	29	47	57	70	71	83
13	2	8	26	36	49	55	62	82
23	9	5	15	28	34	46	61	68
30	20	11	7	19	32	48	51	68
41	32	22	11	11	32	36	52	62
53	43	32	23	12	15	31	41	53
64	53	44	33	22	15	20	32	42
74	65	54	43	31	26	19	21	29
86	75	64	52	47	40	32	25	23

(d)

Fig. 2 The minimum cumulative cost matrix obtained by DP method and tracing backward path
(a) original image gray value matrix (b) the original cost matrix obtained by pre-processing. "3" is the starting point and "2" is the end point (c) cumulative cost matrix after two times iteration (d) minimum cumulative cost matrix after the iteration and tracing backward the least cumulative path to get the contour

图 2 DP 算法获得最小累积代价阵及反向路径跟踪
(a)图像原始灰度矩阵 (b)经过预处理后得到的初始代价阵;"3"为起始点,"2"为终止点
(c)经 2 次迭代后的累积代价阵 (d)迭代完成后获得的最小累积代价阵及反向跟踪最小累积路径得到的边缘线

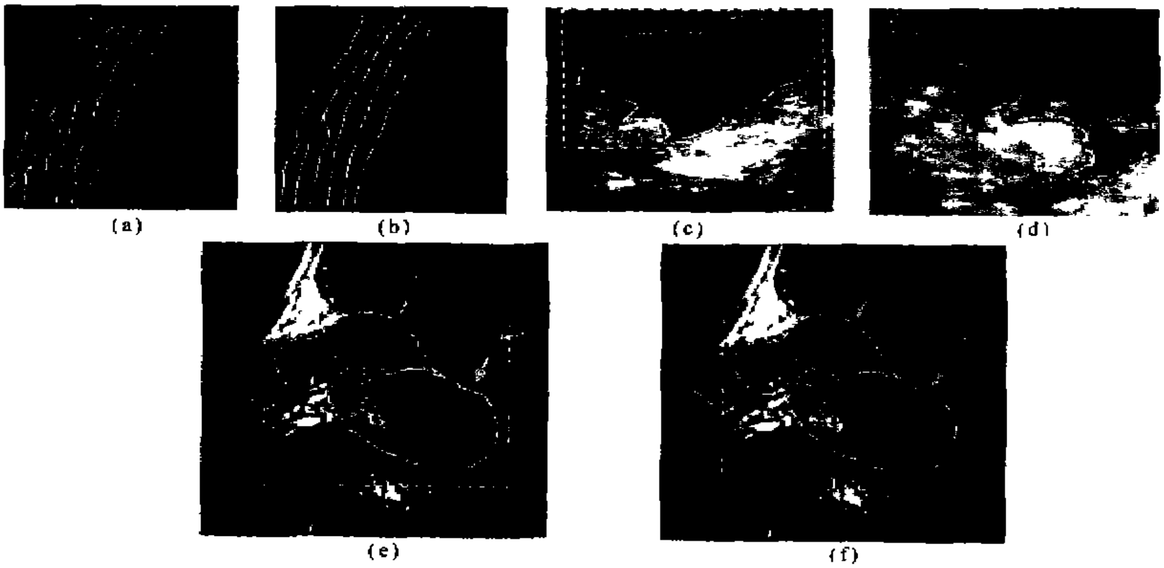

Fig. 3 The contour of images extracted by dynamic programming algorithm and the results embellished by adaptive B-spline

(a) the local results of the fingerprint obtained by dynamic programming (DP) algorithm (b) the fingerprint embellished by adaptive B-spline algorithm (c) an ultrasound image boundary of gastric carcinoma. The contour is detected by DP algorithm and embellished by adaptive B-spline method (d) local magnification of the left image (e) an MR image of the anklebone of foot the contours of the talus and calcaneus extracted by DP algorithm (f) the results of the contours embellished by adaptive cubic B-spline

图 3 各种图像采用动态规划方法提取的轮廓线以及经过 B 样条修饰后的效果

(a) 采用动态规划法提取的指纹图像(局部) (b) 经自适应 B 样条修饰后的指纹纹路 (c) 超声图像上获得的胃癌边缘采用动态规划法提取并经自适应 B 样条修饰 (d) 左图的局部放大图像 (e) 足踝关节的磁共振图像采用动态规划法提取的距骨和跟骨的轮廓线 (f) 轮廓线经自适应 3 次 B 样条修饰后的效果

method. This method combined with image segmentation algorithms can enhance the robustness and the noise-resistibility of the latter. Our experiment indicates that the fitting effect of non-uniform distribution is better than the uniform, if the number of control points is not changed. And the control points number of non-uniform distribution is less than that of the uniform distribution, if the fitting condition is the same. So the iteration time is saved.

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