

Cherenkov 型自由电子激光器增益和效率的研究

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研究了 Cherenkov 型自由电子激光器的增益和效率。结果表明: 在选用合适的物理参量和匹配的工作相位条件下, Cherenkov 型自由电子激光器比普通自由电子激光器有高得多的增益和效率。

关键词: Cherenkov 辐射, 自由电子激光, 增益, 转换效率。

1 引言

自由电子激光(FEL)是当代激光和相对论电子束的前沿课题, 随着一般的 Compton-Raman 自由电子激光器的深入研究, 近几年人们已经转向其它类型自由电子激光器的研究。

由于不同机理的辐射可以在同一实验装置中产生^[1~2], 因此近几年来, 人们对综合利用不同机理辐射的研究很感兴趣, 并取得了一些初步成果^[3, 4]。本文研究的 Cherenkov 型自由电子激光器是综合利用相对论电子束横向动能和部分纵向动能转换成辐射能。在文献[4]中自由电子激光器的 Cherenkov 辐射工作条件的研究基础上本文进一步研究了这种激光器增益和能量转换效率。研究表明, 选用合适的物理参量和匹配的相位情况下, 这种激光器的增益和效率要比一般的自由电子激光器有较大幅度的提高。

2 基本模型与方程

Cherenkov 型自由电子激光器的基本物理模型可概括为以下三大部件: (1) 相对论电子束及其加速器装置; (2) 静态 Wiggler 泵浦磁场系统; (3) 具有 Cherenkov 效应的互作用腔体。这种模型与 Cherenkov CARM 实验装置^[3]较相似, 所不同的是 wiggler 磁场与互作用区不是明显分开。下面我们用单粒子模型理论来分析这种模型的基本方程。

假定相对论电子通过的 wiggler 磁场及其形成的相干辐射场分别为

$$\mathbf{B}_w = B_0(\cos k_0 z \hat{e}_x + \sin k_0 z \hat{e}_y); \quad (1)$$

$$\left. \begin{aligned} \mathbf{E}_r &= E_{r0} [\cos(k_r z - \omega_r t + \psi_r) \hat{e}_x - \sin(k_r z - \omega_r t + \psi_r) \hat{e}_y]; \\ \mathbf{B}_r &= \frac{E_{r0}}{u} [\sin(k_r z - \omega_r t + \psi_r) \hat{e}_x + \cos(k_r z - \omega_r t + \psi_r) \hat{e}_y]; \end{aligned} \right\} \quad (2)$$

式(2)中 E_{r0} 、 k_r 、 ψ_r 分别是相干辐射电场振幅、波矢和初相位。假定由 Cherenkov 效应产生的辐射场为

$$\left. \begin{aligned} \mathbf{E}_c &= E_{c0} [\cos(k_c z - \omega_c t + \psi_c) \hat{e}_x - \sin(k_c z - \omega_c t + \psi_c) \hat{e}_y]; \\ \mathbf{B}_c &= \frac{E_{c0}}{u} [\sin(k_c z - \omega_c t + \psi_c) \hat{e}_x + \cos(k_c z - \omega_c t + \psi_c) \hat{e}_y]; \end{aligned} \right\} \quad (3)$$

式(3)中 E_{c0} 、 k_c 和 ψ_c 分别为 Cherenkov 辐射电场振幅、波矢和初相位, 幅值 E_{c0} 可由推迟势公式解得^[4]; u 为介质系统(或慢波结构)中光波相速,

$$u = \frac{C}{\sqrt{\epsilon\mu}}$$

其中 ϵ 、 μ 分别为系统介电系数和导磁系数。单个相对论电子运动规律由洛伦兹力方程和能量方程表示:

$$\frac{d}{dt}(\gamma \mathbf{v}) = - \frac{|e|}{m_0} (\mathbf{E} + \mathbf{v} \times \mathbf{B}); \quad (4)$$

$$\frac{d}{dt} \gamma = - \frac{|e|}{m_0 C^2} \mathbf{v} \cdot \mathbf{E}; \quad (5)$$

式(4)、(5)中 m_0 为静态电子质量; e 为电子电荷; γ 为相对论能量因子,

$$\gamma = (1 - \beta^2)^{-\frac{1}{2}}, \quad \beta = \frac{v}{C};$$

\mathbf{E} 和 \mathbf{B} 为系统总电场和总磁场, 即 $\mathbf{E} = \mathbf{E}_r + \mathbf{E}_c$, $\mathbf{B} = \mathbf{B}_r + \mathbf{B}_c + \mathbf{B}_w$ 。式(4)右边忽略了电子辐射阻尼力的反作用力项 $\left(\frac{1}{4\pi\epsilon} \frac{2e^2}{3C^3} \frac{d^2\mathbf{v}}{dt^2} \right)$, 这是因为系统中辐射波长远大于电子经典半径。所以辐射阻尼力相对来说很小且可忽略不计^[5], 其反作用力也可忽略不计。由式(4)得无微扰情况下相对论电子运动的三个坐标分量为

$$v_x = \frac{2\Omega_r u}{\omega_r} \sin \xi_{rs} + \frac{2\Omega_c u}{\omega_c} \sin \xi_{cs} - \frac{\Omega_0 u}{\omega_0} \cos k_0 z_s; \quad (6)$$

$$v_y = \frac{2\Omega_r u}{\omega_r} \cos \xi_{rs} + \frac{2\Omega_c u}{\omega_c} \cos \xi_{cs} - \frac{\Omega_0 u}{\omega_0} \sin k_0 z_s; \quad (7)$$

$$\begin{aligned} v_z &= \frac{\Omega_r \Omega_0 u}{\Delta\omega_{0r}} \left(\frac{2}{\omega_r} + \frac{1}{\omega_0} \right) [\sin(\Delta\omega_{0r} t + \theta_r) - \sin \theta_r] \\ &+ \frac{\Omega_c \Omega_0 u}{\Delta\omega_{0c}} \left(\frac{2}{\omega_c} + \frac{1}{\omega_0} \right) [\sin(\Delta\omega_{0c} t + \theta_c) - \sin \theta_c] \\ &+ \frac{2\Omega_r \Omega_c u}{\Delta\omega_{rc}} \left(\frac{1}{\omega_r} - \frac{1}{\omega_c} \right) [\cos(\Delta\omega_{rc} t + \theta_{rc}) - \cos \theta_{rc}]; \end{aligned} \quad (8)$$

式(6)和(7)中

$$\Omega_r = \frac{|e| E_{r0}}{\gamma_s m_0 u}, \quad \Omega_c = \frac{|e| E_{c0}}{\gamma_s m_0 u}, \quad \Omega_0 = \frac{|e| B_0}{\gamma_s m_0};$$

$$\xi_{rs} = k_r z_s - \omega_r t + \psi_r, \quad \xi_{cs} = k_c z_s - \omega_c t + \psi_c; \quad z_s = z_0 + v_z^0 t;$$

其中 γ_s 是电子与波相互作用前相对论因子, $\omega_0 = k_0 u$, $\omega_r = k_r u$, $\omega_c = k_c u$, v_z^0 是相对论电子纵向初速度。式(8)中

$$\left. \begin{aligned} \Delta\omega_{0r} &= (k_r + k_0)v_s^0 - \omega_r, \quad \theta_r = (k_r + k_0)z_0 + \psi_r; \\ \Delta\omega_{0c} &= (k_c + k_0)v_s^0 - \omega_c, \quad \theta_c = (k_c + k_0)z_0 + \psi_c; \\ \Delta\omega_{rc} &= (k_r - k_c)v_s^0 - (\omega_r - \omega_c), \quad \theta_{rc} = (k_r - k_c)z_0 + \psi_r - \psi_c; \\ \Delta\omega_{r0} &= \Delta\omega_{0r} - \Delta\omega_{0c}, \quad \theta_{r0} = \theta_r - \theta_c. \end{aligned} \right\} \quad (9)$$

3 增益与分析

由于相对论电子运动中受到辐射场的微扰作用,其运动轨迹和能量都将发生微扰变化,即

$$\gamma = \gamma_s + \delta\gamma. \quad (10)$$

$$z = z_s + \delta z, \quad x = x_s + \delta x, \quad y = y_s + \delta y. \quad (11)$$

利用近似公式 $\sin(x + \delta x) \approx \sin x + \delta x \cos x$, $\cos(x + \delta x) \approx \cos x - \delta x \sin x$, 得微扰作用后的电场和电子横向速度为

$$\mathbf{E}_r \approx E_{r0} [(\cos \xi_{rs} - k_r \delta z \sin \xi_{rs}) \hat{e}_x - (\sin \xi_{rs} + k_r \delta z \cos \xi_{rs}) \hat{e}_y]; \quad (12)$$

$$\mathbf{E}_c \approx E_{c0} [(\cos \xi_{cs} - k_c \delta z \sin \xi_{cs}) \hat{e}_x - (\sin \xi_{cs} + k_c \delta z \cos \xi_{cs}) \hat{e}_y]; \quad (13)$$

$$\begin{aligned} v_x \approx & \frac{2\Omega_r u}{\omega_r} (\sin \xi_{rs} + k_r \delta z \cos \xi_{rs}) + \frac{2\Omega_c u}{\omega_c} (\sin \xi_{cs} + k_c \delta z \cos \xi_{cs}) \\ & - \frac{\Omega_0 u}{\omega_0} (\cos k_0 z_s - k_0 \delta z \sin k_0 z_s); \end{aligned} \quad (14)$$

$$\begin{aligned} v_y \approx & \frac{2\Omega_r u}{\omega_r} (\cos \xi_{rs} - k_r \delta z \sin \xi_{rs}) + \frac{2\Omega_c u}{\omega_c} (\cos \xi_{cs} - k_c \delta z \sin \xi_{cs}) \\ & - \frac{\Omega_0 u}{\omega_0} (\sin k_0 z_s + k_0 \delta z \cos k_0 z_s). \end{aligned} \quad (15)$$

设 $\delta\gamma = \delta\gamma_1 + \delta\gamma_2$, 由式(5)得

$$\frac{d}{dt} \gamma = \frac{d}{dt} \delta\gamma_1 + \frac{d}{dt} \delta\gamma_2 = - \frac{|e|}{m_0 c^2} (v_x E_x + v_y E_y); \quad (16)$$

其中

$$\begin{aligned} \frac{d}{dt} \delta\gamma_1 = & \frac{\Omega_0 \Omega_r u^2 \gamma_s}{C^2 \omega_0} \cos(\Delta\omega_{0r} t + \theta_r) + \frac{\Omega_0 \Omega_c u^2 \gamma_s}{C^2 \omega_0} \cos(\Delta\omega_{0c} t + \theta_c) \\ & - \frac{2\Omega_r \Omega_c u^2 \gamma_s}{C^2} \left(\frac{1}{\omega_r} + \frac{1}{\omega_c} \right) \sin(\Delta\omega_{rc} t + \theta_{rc}); \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{d}{dt} \delta\gamma_2 = & - \frac{\Omega_0 \Omega_r u^2 \gamma_s (k_0 + k_s)}{C^2 \omega_0} \sin(\Delta\omega_{0r} t + \theta_r) \delta z - \frac{\Omega_0 \Omega_c u^2 \gamma_s (k_0 + k_c)}{C^2 \omega_0} \sin(\Delta\omega_{0c} t + \theta_c) \delta z \\ & - \frac{2\Omega_r \Omega_c u^2 \gamma_s (k_r - k_c)}{C^2} \left(\frac{1}{\omega_r} - \frac{1}{\omega_c} \right) \cos(\Delta\omega_{rc} t + \theta_{rc}) \delta z. \end{aligned} \quad (18)$$

自由电子激光增益可表示为^[8]

$$G = - \langle \delta\gamma \rangle_{\psi_r, \psi_c} m_0 u^2 4 \pi \rho_e V / \varepsilon E_0^2. \quad (19)$$

式中 ρ_e 是电子束密度, V 为相互作用区体积, $\langle \delta\gamma \rangle_{\psi_r, \psi_c}$ 为电子跃变能量对辐射场初相位 ψ_r 、 ψ_c 的平均值, 即

$$\langle \delta\gamma \rangle_{\psi_r, \psi_c} = \langle \delta\gamma_1 \rangle_{\psi_r, \psi_c} + \langle \delta\gamma_2 \rangle_{\psi_r, \psi_c}. \quad (20)$$

由式(17)对 t 积分得

$$\begin{aligned} \delta\gamma_1 = & \frac{\Omega_0\Omega_r u^2 \gamma_s}{C^2 \omega_0 \Delta\omega_{0r}} [\sin(\Delta\omega_{0r}t + \theta_r) - \sin\theta_r] + \frac{\Omega_0\Omega_c u^2 \gamma_s}{C^2 \omega_0^2 \Delta\omega_{0c}} [\sin(\Delta\omega_{0c}t + \theta_c) - \sin\theta_c] \\ & + \frac{2\Omega_r\Omega_c u^2 \gamma_s}{C^2 \Delta\omega_{rc}} \left(\frac{1}{\omega_r} + \frac{1}{\omega_c} \right) [\cos(\Delta\omega_{rc}t + \theta_{rc}) - \cos\theta_{rc}]; \end{aligned} \quad (21)$$

所以

$$\langle \delta\gamma_1 \rangle_{\psi_r \psi_c} = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \delta\gamma_1 d\psi_r d\psi_c = 0. \quad (22)$$

式(18)表明要求 $\delta\gamma_2$ 必须先解得 δz , 为此我们利用 $\frac{dz}{dt} = v_z$ 关系式, 由式(8)对 t 积分, 且结合式(11)中 $z = z_s + \delta z$ 的关系, 得

$$\begin{aligned} \delta z = & \frac{\Omega_0\Omega_r u}{\Delta\omega_{0r}^2} \left(\frac{2}{\omega_r} + \frac{1}{\omega_0} \right) [\cos\theta_r - \cos(\Delta\omega_{0r}t + \theta_r) - \Delta\omega_{0r}t \sin\theta_r] \\ & + \frac{\Omega_0\Omega_c u}{\Delta\omega_{0c}^2} \left(\frac{2}{\omega_c} + \frac{1}{\omega_0} \right) [\cos\theta_c - \cos(\Delta\omega_{0c}t + \theta_c) - \Delta\omega_{0c}t \sin\theta_c] \\ & + \frac{2\Omega_r\Omega_c u}{\Delta\omega_{rc}^2} \left(\frac{1}{\omega_r} - \frac{1}{\omega_c} \right) [\sin(\Delta\omega_{rc}t + \theta_{rc}) - \sin\theta_{rc} - \Delta\omega_{rc}t \cos\theta_{rc}] - z_s. \end{aligned} \quad (23)$$

式(23)代入式(18)并设 $\delta\gamma_2 = \delta\gamma_2^{(1)} + \delta\gamma_2^{(2)}$, 则有

$$\begin{aligned} \delta\gamma_2^{(1)} = & \frac{\Omega_0\Omega_r u^2 \gamma_s (k_0 + k_r)}{C^2 \omega_0 \Delta\omega_{0r}} \left\{ z_0 [\cos\theta_r - \cos(\Delta\omega_{0r}t + \theta_r)] \right. \\ & \left. + \frac{v_z^0}{\Delta\omega_{0r}} [\sin(\Delta\omega_{0r}t + \theta_r) - \sin\theta_r - \Delta\omega_{0r}t \cos(\Delta\omega_{0r}t + \theta_r)] \right\} \\ & + \frac{\Omega_0\Omega_c u^2 \gamma_s (k_0 + k_c)}{C^2 \omega_0 \Delta\omega_{0c}} \left\{ z_0 [\cos\theta_c - \cos(\Delta\omega_{0c}t + \theta_c)] \right. \\ & \left. + \frac{v_z^0}{\Delta\omega_{0c}} [\sin(\Delta\omega_{0c}t + \theta_c) - \sin\theta_c - \Delta\omega_{0c}t \cos(\Delta\omega_{0c}t + \theta_c)] \right\} \\ & + \frac{2\Omega_r\Omega_c u^2 \gamma_s (k_r - k_c)}{C^2 \Delta\omega_{rc}} \left(\frac{1}{\omega_r} - \frac{1}{\omega_c} \right) \left\{ z_0 [\sin(\Delta\omega_{rc}t + \theta_{rc}) - \sin\theta_{rc}] \right. \\ & \left. - \frac{v_z^0}{\Delta\omega_{rc}} [\cos\theta_{rc} - \cos(\Delta\omega_{rc}t + \theta_{rc}) - \Delta\omega_{rc}t \sin(\Delta\omega_{rc}t + \theta_{rc})] \right\}; \end{aligned} \quad (24)$$

所以得

$$\langle \delta\gamma_2^{(1)} \rangle_{\psi_r \psi_c} = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \delta\gamma_2^{(1)} d\psi_r d\psi_c = 0. \quad (25)$$

同理可解得 $\langle \delta\gamma_2^{(2)} \rangle_{\psi_r \psi_c}$ 为

$$\begin{aligned} \langle \delta\gamma_2^{(2)} \rangle_{\psi_r \psi_c} = & -\frac{u \Sigma_{0r} H_{0r}}{C \omega_0 \Delta\omega_{0r}^2} \left(\frac{2}{\omega_r} + \frac{1}{\omega_0} \right) \left[1 - \cos\Delta\omega_{0r}t - \frac{1}{2} \Delta\omega_{0r}t \sin\Delta\omega_{0r}t \right] \\ & - \frac{u \Sigma_{0c} H_{0c}}{C \omega_0 \Delta\omega_{0c}^2} \left(\frac{2}{\omega_c} + \frac{1}{\omega_0} \right) \left[1 - \cos\Delta\omega_{0c}t - \frac{1}{2} \Delta\omega_{0c}t \sin\Delta\omega_{0c}t \right] \\ & - \frac{4u \Sigma_{rc} H_{rc}}{C \Delta\omega_{rc}} \left(\frac{1}{\omega_r} - \frac{1}{\omega_c} \right)^2 \left[1 - \cos\Delta\omega_{rc}t - \frac{1}{2} \Delta\omega_{rc}t \sin\Delta\omega_{rc}t \right]; \end{aligned} \quad (26)$$

其中

$$\left. \begin{aligned} \Sigma_{0r} &= (k_0 + k_r) \Omega_0 \Omega_r \gamma_s u / C, \quad H_{0r} = \Omega_0 \Omega_r / \Delta\omega_{0r}; \\ \Sigma_{0c} &= (k_0 + k_c) \Omega_0 \Omega_c \gamma_s u / C, \quad H_{0c} = \Omega_0 \Omega_c / \Delta\omega_{0c}; \\ \Sigma_{rc} &= (k_r - k_c) \Omega_r \Omega_c \gamma_s u / C, \quad H_{rc} = \Omega_r \Omega_c / \Delta\omega_{rc}. \end{aligned} \right\} \quad (27)$$

由式(26)、(25)和(22)得增益

$$G = g \left\{ \frac{\Omega_0^2 \Omega_r^2 (k_0 + k_r)}{\omega_0 \Delta \omega_{0r}^3} \left(\frac{2}{\omega_r} + \frac{1}{\omega_0} \right) \left[-\cos \Delta \omega_{0r} t - \frac{1}{2} \Delta \omega_{0r} t \sin \Delta \omega_{0r} t \right] \right. \\ + \frac{\Omega_0^2 \Omega_c^2 (k_0 + k_c)}{\omega_0 \Delta \omega_{0c}^3} \left(\frac{2}{\omega_c} + \frac{1}{\omega_0} \right) \left[1 - \cos \Delta \omega_{0c} t - \frac{1}{2} \Delta \omega_{0c} t \sin \Delta \omega_{0c} t \right] \\ \left. + \frac{4 \Omega_r^2 \Omega_c^2 (k_r - k_c)}{\Delta \omega_{rc}^3} \left(\frac{1}{\omega_r} - \frac{1}{\omega_c} \right)^2 \left[1 - \cos \Delta \omega_{rc} t - \frac{1}{2} \Delta \omega_{rc} t \sin \Delta \omega_{rc} t \right] \right\}; \quad (28)$$

式中 $g = \frac{8\pi m_0 \gamma_s \rho_e \mu^3}{s C^2 E_0^2}$. 式(28)表明 Cherenkov 型自由电子激光器增益由三部分组成: (1) 电子束相干辐射增益 G_r ; (2) Cherenkov 辐射增益 G_c ; (3) 二种辐射相互作用增益 G_{rc} (式(28)中右边第 3 项). 由于系统满足 Cherenkov 辐射条件^[4], 所以有

$$\Delta \omega_{rc} = (\omega_r - \omega_c) \left[\frac{v_z^0}{u} - 1 \right] \geq 0 \quad (\text{当 } \omega_r \geq \omega_c \text{ 时}) \quad (29)$$

显然有 $G > G_r$, 可见激光器参数在合适情况下, Cherenkov 型自由电子激光器增益比仅有相干辐射的自由电子激光器增益要高得多 (见图 1).

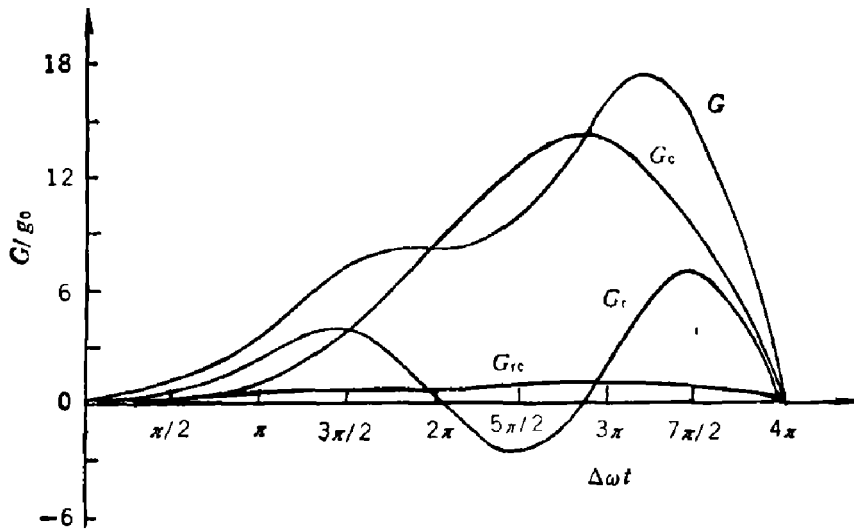


图 1 当参量满足 $\omega_r = 10^4 \omega_0$, $\omega_c = \frac{1}{2} \omega_r$; 且假定 $E_{c0} \approx E_{r0} \approx \frac{1}{2} E_0$, $B_0 \approx E_0/u$ 时计算所得 G 和单程 G_r, G_c, G_{rc} 之间对应曲线

Fig. 1 When the parameters are $\omega_r = 10^4 \omega_0$, $\omega_c = \frac{1}{2} \omega_r$, and let $E_{c0} \approx E_{r0} \approx \frac{1}{2} E_0$, $B_0 \approx E_0/u$, the relations curves between calculating gain G and simplex G_r, G_c, G_{rc}

如果 $\omega_r < \omega_c$, 则表明 G_{rc} 的系数为负值, G 将要减小, 所以 Cherenkov 型自由电子激光器应当选择在 $\omega_r \geq \omega_c$ 的工作条件下才能获得较高增益输出.

如果 Cherenkov 效应不满足, 则 $E_{c0} = 0$, 显然有 $G_c = G_{rc} = 0$, 由式(28)可得一般真空情况下 ($u = c$) 自由电子激光器增益表达式

$$G_r = \frac{4e^4 B_0^2 \rho_e \lambda_0 \left(1 + \frac{\lambda_0}{\lambda_r} \right)}{(m_0 \gamma_s C \Delta \omega_{0r})^3} \left[1 - \cos \Delta \omega_{0r} t - \frac{1}{2} \Delta \omega_{0r} t \sin \Delta \omega_{0r} t \right] \quad (30)$$

式(30)与文献[6]所得的结果是基本一致的.

4 效率与讨论

自由电子激光器是把相对论电子束部分动能转换成辐射能. 假定系统中相对论电子无纵向电场加速, 并且系统中也无能量损耗, 则激光器的能量转换效率可表示为

$$\eta = 1 - \frac{v_z^2}{v_{z0}^2}; \quad (31)$$

式(31)中 v_{z0} 为入射相对论电子纵向速度, 在入射电子束无发散情况下, 则

$$v_{z0} = v_0 = C(\gamma_0^2 - 1)^{1/2} / \gamma_0; \quad (32)$$

$$v_z^2 = v^2 - v_{\perp}^2 = C^2(\gamma^2 - 1) / \gamma^2 - v_{\perp}^2; \quad (33)$$

式(33)中 v_{\perp} 是在互作用区内相对论电子的横向速度, 由式(6)和(7)得

$$v_{\perp}^2 = u^2 \left[\left(\frac{2\Omega_r}{\omega_r} \right)^2 + \left(\frac{2\Omega_c}{\omega_c} \right)^2 + \left(\frac{\Omega_0}{\omega_0} \right)^2 + \frac{8\Omega_r\Omega_c}{\omega_r\omega_c} \cos(\Delta\omega_{rc}t + \theta_{rc}) - \frac{4\Omega_r\Omega_0}{\omega_r\omega_0} \sin(\Delta\omega_{or}t + \theta_r) - \frac{4\Omega_c\Omega_0}{\omega_c\omega_0} \sin(\Delta\omega_{oc}t + \theta_c) \right]; \quad (34)$$

则效率 η 为

$$\eta = 1 - \frac{\gamma_0^2}{(\gamma_0^2 - 1)\gamma^2} \left\{ \gamma^2 - 1 - \frac{\gamma^2 e^2}{C^2 \gamma_s^2 m_0^2} \left[\frac{4E_{r0}^2}{\omega_r^2} + \frac{4E_{c0}^2}{\omega_c^2} + \frac{B_0^2 u^2}{\omega_0^2} + \frac{8E_{r0}E_{c0}}{\omega_r\omega_c} \cos(\Delta\omega_{rc}t + \theta_{rc}) - \frac{4E_{r0}B_0 u}{\omega_r\omega_0} \sin(\Delta\omega_{or}t + \theta_r) - \frac{4E_{c0}B_0 u}{\omega_c\omega_0} \sin(\Delta\omega_{oc}t + \theta_c) \right] \right\}; \quad (35)$$

式(35)中 $\gamma \leq \gamma_s \leq \gamma_0$, γ_s , γ 分别是微扰前和微扰后相对论电子能量因子. 由式(35)可得真空情况下一般自由电子激光器效率

$$\eta_0 = 1 - \frac{\gamma_0^2}{(\gamma_0 - 1)\gamma^2} \left\{ \gamma^2 - 1 - \frac{\gamma^2 e^2}{C^2 \gamma_s^2 m_0^2} \left[\frac{4E_{r0}^2}{\omega_r^2} + \frac{B_0^2 C^2}{\omega_0^2} - \frac{4E_{r0}B_0 C}{\omega_r\omega_0} \sin(\Delta\omega_{or}t + \theta_r) \right] \right\} \quad (36)$$

式(36)说明, 激光器效率不仅与 γ_0 , B_0 , w_r 等参量有关, 而且与激光器工作相位匹配状况有重要关系. 如果工作相位处于最佳匹配情况, 即 $\sin(\Delta\omega_{or}t + \theta_r) \approx -1$, 由式(36)可见, 则效率达到最大值, 这相当于自由电子激光器实验中输出辐射功率达到饱和状态.

由式(35)和(36)可得两类效率差为

$$\Delta\eta = \eta - \eta_0 = \frac{\gamma_0^2}{\gamma_0^2 - 1} \frac{e^2}{C^2 \gamma_s^2 m_0^2} \left[\frac{4E_{c0}^2}{\omega_c^2} + \frac{B_0^2 (u^2 - C^2)}{\omega_0^2} + \frac{8E_{r0}E_{c0}}{\omega_r\omega_c} \cos(\Delta\omega_{rc}t + \theta_{rc}) - \frac{4E_{c0}B_0 u}{\omega_c\omega_0} \sin(\Delta\omega_{oc}t + \theta_c) - \frac{4E_{r0}B_0 (u - C)}{\omega_r\omega_0} \sin(\Delta\omega_{or}t + \theta_r) \right]. \quad (37)$$

从式(37)不难看到, 如果 Cherenkov 辐射的工作相位也处于最佳匹配状态 ($\sin(\Delta\omega_{oc}t + \theta_c) \approx -1$), 则有 $\Delta\eta > 0$. 这表明: Cherenkov 型自由电子激光器在合适的工作相位和物量参量情况下, 可获得比普通型自由电子激光器高得多的效率.

5 结束语

本文分析的 Cherenkov 型自由电子激光器是一类集两种不同辐射机理于一体的综合

性激光器,它与近几年国外发展起来的 Cherenkov CARM^[3]十分相似。综合利用相对论电子束的纵向和横向动能转换成辐射能的装置显然要比只利用横向动能转换成辐射能的一般自由电子激光器要优越。本文分析了 Cherenkov 型自由电子激光器增益和效率,表明在合理的物理参量情况下它比一般自由电子激光器有较大幅度提高,这恰好说明这种激光器优越之处。我们认为,随着激光器的发展,Cherenkov 型自由电子激光器将可成为一类高增益和高效型自由电子激光器。

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STUDIES ON GAIN AND EFFICIENCY OF CHERENKOV FEL

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The gain and efficiency of Cherenkov FEL are studied. The results show that under the conditions of suitable physical parameters and matching operating phase, the gain and efficiency of Cherenkov FEL are considerably higher than those of common free electron lasers.

Key words: Cherenkov radiation, free-electron laser, gain, transformation efficiency.