

## Thermal effect on odd-symmetric phase mask in wavefront-coded athermalized infrared imaging systems

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**Abstract:** Thermal effect on phase masks in wavefront-coded athermalized imaging systems is analyzed. Several well-known odd-symmetric phase functions at different temperature are derived. The properties of phase functions suffering from thermal effect are presented. The performances of the phase functions are evaluated with and without considering thermal effect by numerical simulation. The results show that the similarity of out-of-focus MTFs and recoverability of blurred encoding images of the wavefront-coded athermalized system are degraded severely by the thermal effect.

**Key words:** wavefront coding; athermalization; thermal effect; phase mask

**PACS:**42.15.Dp, 42.15.Fr, 42.88.+h

## 基于波前编码的红外无热化光学系统相位掩模板热效应特性分析

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**摘要:** 分析了基于波前编码技术的无热化光学系统中相位掩模板的自身热效应, 得到了不同温度下光瞳处相位板相位函数的变化. 分析了由相位板热效应引起的波前编码系统成像特性的变化. 通过数值及图像仿真证明了相位板热效应对波前编码无热化光学系统性能产生的影响. 结果表明, 相位板热效应对系统 MTF(调制传递函数)的离焦不变特性和编码图像的可复原性产生严重的影响.

**关键词:** 波前编码; 无热化; 热效应; 相位掩模板

**中图分类号:** O435.2; O438.2 **文献标识码:** A

### Introduction

The performance of infrared optical systems applied in the thermal environment changes with temperatures. Main thermal effect on infrared optical systems in a wide temperature range is the defocus aberration. It is inevitable to resort to athermalized design in order to maintain an invariable focus of infrared optical systems at different ambient temperatures<sup>[1]</sup>. Recently, the wavefront coding technique is used to reduce thermal defocus aberrations of infrared optical systems<sup>[2-3]</sup>. As an optical-digital hybrid imaging system, wave-front coding is used to extend the depth of field of the incoherent diffraction-limited systems. In wavefront coding systems, an odd-symmetrical phase

mask is placed in the pupil plane to encode the wavefront of incident light waves so that the optical transfer function (OTF) or point spread function (PSF) is nearly invariant over a wide range of defocus. The encoded images are subsequently deblurred with a simple deconvolution filter. The restored images retain high resolution near that of the diffraction-limited system for all values of defocus<sup>[4]</sup>.

Generally, the optimizing procedure of phase mask parameter is not included in the optical design. The thermal effect on phase mask in the wavefront-coded athermalized systems is ignored due to the separate optimization step<sup>[5-6]</sup>. Specifically, the evaluation of designed phase mask parameters in the optical design software (ZEMAX, CODE V etc.) is inaccurate.

rate because the environmental analysis (ENV) option of the optical design software is designed to operate with rotationally symmetric surfaces. The inexact PSFs can degrade the quality of final images. More importantly, a comprehensive and rigorous evaluation of the performance of wavefront coding systems is indispensable in order to ensure the qualities of restored images. Thus, the thermal effect on phase mask in the wavefront-coded athermalized systems should be considered in the mask parameter's optimization procedure in order to obtain the accurate PSFs at different temperatures in theory. In this paper, we propose the phase functions resulted from a contrast between two types of phase masks with and without thermal effect considered. The accurate OTF taking into account thermal effect on phase masks is obtained to build the deconvolution filter.

## 1 Theory

For mathematical simplicity yet without loss of generality, the analysis is restricted to the case of one-dimensional optical system with a rectangularly separable aperture. The refractive index, center thickness and surface shape of phase masks change with temperature. Because the center thickness does not affect phase distribution of phase masks in pupil plane, we concentrate only on the changes of phase masks' refractive index and surface shape.

For an optical system with an odd-symmetric phase function  $\Phi(u)$  ( $u$  is the pupil plane coordinate and is normalized to the range of  $[-1, 1]$ ), when the temperature changes from  $t$  to  $t_c = t + \Delta t$ , the pupil coordinate can be expressed as

$$u_c = u + du = u(1 + \alpha_0 \Delta t) \quad , \quad (1)$$

where  $u_c$  is the pupil plane coordinate in the thermal environment,  $\alpha_0$  is the thermal expansion coefficient of the material, and  $du = u\alpha_0 \Delta t$  is variation of pupil coordinate. The change in surface shape of phase mask is

$$d\Phi(u) = [\partial\Phi(u)/\partial u] du \quad . \quad (2)$$

The surface shape of phase mask in the thermal environment is given by

$$\Phi_{\text{shape-change}}(u) = \Phi(u) + d\Phi(u) \quad . \quad (3)$$

Additionally, the relationship between the phase and the optical path is

$$\Phi(u) = \frac{2\pi}{\lambda} H(u) = \frac{2\pi}{\lambda} (n-1)d(u) \quad , \quad (4)$$

where  $\lambda$  is the wavelength of light,  $n$  is the phase mask's refractive index,  $d(u)$  is the phase mask's

thickness (in unit of millimeter), and  $H(u)$  is the optical path difference. At temperature  $t_c$ , the refractive index in the thermal environment is

$$n_c = n + (dn/dt)\Delta t \quad , \quad (5)$$

where  $dn/dt$  is the thermal refractive index coefficient. For the refractive index  $n_c$ , the phase function,  $\Phi_{\text{refractive-index-change}}(u)$ , can be expressed as

$$\Phi_{\text{refractive-index-change}}(u) = \frac{2\pi}{\lambda} (n_c - 1)d(u) \quad . \quad (6)$$

From Eqs. (4) and (6), the phase function is

$$\Phi_{\text{refractive-index-change}}(u) = [(n_c - 1)/(n - 1)]\Phi(u) \quad . \quad (7)$$

Obviously, the changes in both refractive index and surface shape produce the phase deviations in the pupil plane. Thus, the phase function in the thermal environment,  $\Phi_t(u)$ , can be expressed as

$$\Phi_t(u) = \frac{n_c - 1}{n - 1} \Phi_{\text{shape-change}}(u) \quad . \quad (8)$$

In this paper, the cubic phase mask (CPM), the logarithmic phase mask (LPM), the exponential phase mask (EPM) and the sinusoidal phase mask (SPM) are discussed as examples<sup>[4, 7-9]</sup>. The four phase functions can be expressed as

$$\Phi_{\text{CPM}}(u) = Au^3 \quad , \quad (9)$$

$$\Phi_{\text{LPM}}(u) = \text{sgn}(u)Bu^2(\log|u| + C) \quad , \quad (10)$$

$$\Phi_{\text{EPM}}(u) = Du\exp(Eu^2) \quad , \quad (11)$$

$$\Phi_{\text{SPM}}(u) = Fu^4\sin(Gu) \quad , \quad (12)$$

where  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $F$ , and  $G$  are the magnitude of the variable phase parameters, and  $\text{sgn}(\cdot)$  is the signum function.

From Eqs. (8-12), four phase functions in the thermal environment are

$$\Phi_{t\text{CPM}} = \frac{n_c - 1}{n - 1} (A + 3\alpha_0 A \Delta t) u^3 \quad , \quad (13)$$

$$\Phi_{t\text{LPM}} = \frac{n_c - 1}{n - 1} \text{sgn}(u) [(B + 2\alpha_0 B \Delta t) u^2 (\log|u| + C) + \alpha_0 B \Delta t u^3 |u|^{-1}] \quad , \quad (14)$$

$$\Phi_{t\text{EPM}} = \frac{n_c - 1}{n - 1} [(D + \alpha_0 D \Delta t) u \exp(Eu^2) + 2\alpha_0 D E \Delta t u^3 \exp(Eu^2)] \quad , \quad (15)$$

$$\Phi_{t\text{SPM}} = \frac{n_c - 1}{n - 1} [(F + 4\alpha_0 F \Delta t) u^4 \sin(Gu) + \alpha_0 F G \Delta t u^5 \cos(Gu)] \quad . \quad (16)$$

Eqs. (13-16) are functions of temperature and represent temperature-dependent phase deviations of phase masks instead of phase deviations at different temperature.

## 2 Numerical examples

All contraries to our approach were mainly a-

chieved from thermal defocus-aberration considerations without including other thermal aberrations. The similarity of out-of-focus MTF (Modulation Transfer Function) and recoverability of blurred encoding images can be used to assess the performance of wavefront coding systems. We discuss the original phase masks of Eqs. (9-12) (without thermal effect considered) and the actual phase masks of Eqs. (13-16) (with thermal effect considered) by the properties of MTFs. We use the Hilbert space angles, the Fisher information content and the integral area of MTF to compare the imaging characteristics between the phase masks with and without thermal effect considered.

Considering the classical infrared material germanium as an example, the refractive index  $n$  for the center wavelength  $10 \mu\text{m}$  is 4.003, the thermal refractive index coefficient  $dn/dt$  is  $3.96 \times 10^{-4}$  per degree Celsius, and the thermal expansion coefficient  $\alpha_0$  is  $5.9 \times 10^{-6}$  per degree Celsius<sup>[10]</sup>. The temperature range  $\Delta t$  is  $-70$  degrees Celsius (A contrast that the system works at  $+20$  degrees Celsius and at  $-50$  degrees Celsius, and  $+20$  degrees Celsius is the original temperature environment).

The Hilbert space angle is a convenient mathematical tool to measure the similarity between any two functions<sup>[7]</sup>. The Hilbert space angle between the in-focus MTF and the defocused MTF is expressed as

$$\cos\theta = \frac{\langle MTF(v,0), MTF(v,\omega_{20}) \rangle}{\|MTF(v,0)\| \|MTF(v,\omega_{20})\|}, \quad (17)$$

where the symbol  $\langle \cdot \rangle$  denotes the inner product, the symbol  $\| \cdot \|$  denotes the norm.  $\omega_{20}$  is the defocus parameter in unit of wavelength  $\lambda$  and  $v$  is the normalized spatial frequency coordinate. The smaller the angle between the two MTFs is, the more similar these two MTFs are. The Hilbert space angles between the in-focus MTF and the defocused MTFs are shown in Fig. 1.

By Rayleigh rule, the image quality of an optical system is acceptable when  $\omega_{20} \leq \lambda/4$ . In the paper, we assume that the maximum defocus caused by thermal effect on the optical systems is  $5\lambda$ . The subplot in the logarithmic phase mask of Fig. 1 is a partial contrast of Hilbert space angle between the actual ( $-50$  degrees Celsius) and original ( $+20$  degrees Celsius) phase masks. All the optimized masks' parameters come from Refs. [9] and [11]. The performances of four masks are evaluated by Fisher Information matrixes of out-of-focus OTFs, and the optimization procedure subjects to the same constraint of

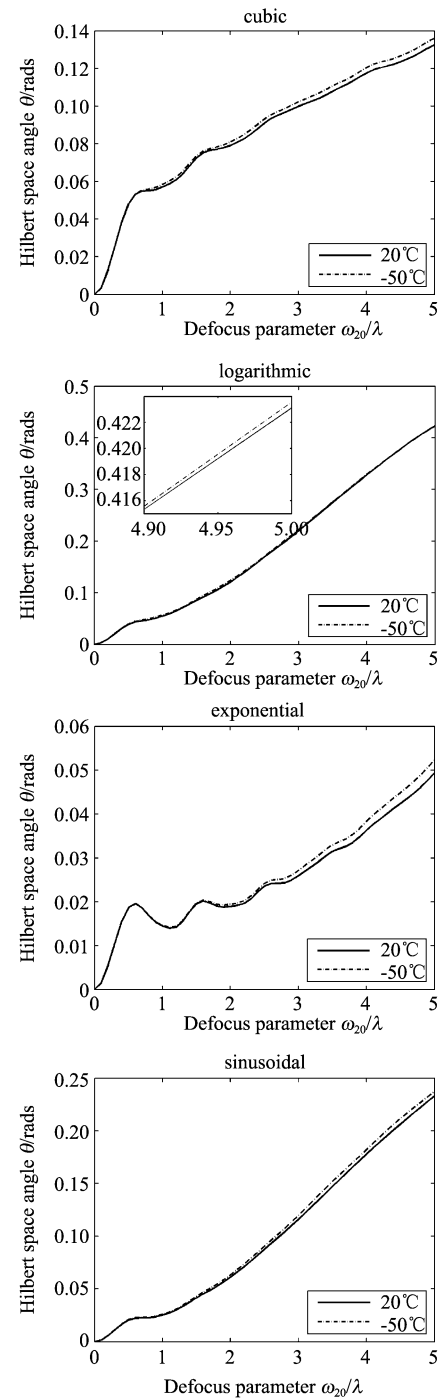


Fig. 1 The Hilbert space angles between the in-focus MTF and the defocused MTFs for the original and actual phase masks. The optimized phase mask parameters are used: CPM;  $A = 90.1096$ ; LPM;  $B = 144.6175$ , and  $C = 0.5866$ ; EPM;  $D = 31.0272$  and  $E = 1.7477$ ; SPM;  $F = 192.9951$  and  $G = 1.7840$ <sup>[9, 11]</sup>. The phase mask parameters of other figures in this paper are the same as these used in Fig. 1

图1 考虑相位板热效应前后由正焦 MTF 和离焦 MTF 构成的 Hilbert 空间角随离焦参数的变化. 相位掩模板参数分别为: CPM:  $A = 90.1096$ ; LPM:  $B = 144.6175$ ,  $C = 0.5866$ ; EPM:  $D = 31.0272$ ,  $E = 1.7477$ ; SPM:  $F = 192.9951$ ,  $G = 1.7840$ <sup>[9, 11]</sup>. 本文中其它图片的相位板参数与图1相同

the acceptable degradation of the in-focus MTF. As is shown in Fig. 1, the Hilbert space angles of the actual phase masks are different from the original ones, especially when there is severe defocus. This means that a phase deviation from the original phase distribution will appear in the pupil plane when the thermal effect on phase masks is considered. Additionally, according to the meaning of Hilbert space angle, the phase masks with thermal effect are much more sensitive to defocus aberrations. The reason is that the negative  $\Delta t$  will decrease the amount of the phase deviations in the pupil plane, indicating that the sensitivity of the MTFs to defocus aberrations increases. But in the positive  $\Delta t$  situation the results are reversed.

Another quantitative way to evaluate the defocus invariant characteristic for phase masks is Fisher Information (FI)<sup>[8]</sup>. According to the definition of FI, the smaller FI is, the less sensitive to defocus the wavefront coding system should be. The FI curves corresponding to different defocus parameters are shown in Fig. 2. It is obvious that there is a considerable change of defocus invariant characteristic between the actual phase masks and the original phase masks. The bigger changes of FI for each phase mask will increase the difference between the original PSFs (OTFs) and the actual PSFs (OTFs). The accurate PSFs or OTFs are prerequisite conditions to ensure higher quality of final images in the wavefront coding systems. Thus, it is reasonable to consider the thermal effect on phase masks in the wavefront-coded athermalized imaging systems in order to obtain the accurate PSFs or OTFs.

For the wavefront coding systems, the magnitude of MTF has to be sacrificed in order to extend large depth of field. The integral area of MTF corresponding to the defocus parameters might be used to assess the middle images' recoverability. The integral area  $S$  can be expressed as

$$S = \int MTF(v, PMP, \omega_{20}) dv, \quad (18)$$

where PMP is the phase mask parameter. The areas of MTFs for each phase mask are shown in Fig. 3. The visible differences in the area of MTF are observed between the actual phase masks and the original phase masks, indicating that the thermal effect on phase masks introduces an obvious phase deviation from the original phase distribution in the pupil plane. The smaller areas of MTFs for the original phase masks comparing with the actual phase masks are found in Fig. 3. This means that the actual phase masks' phase

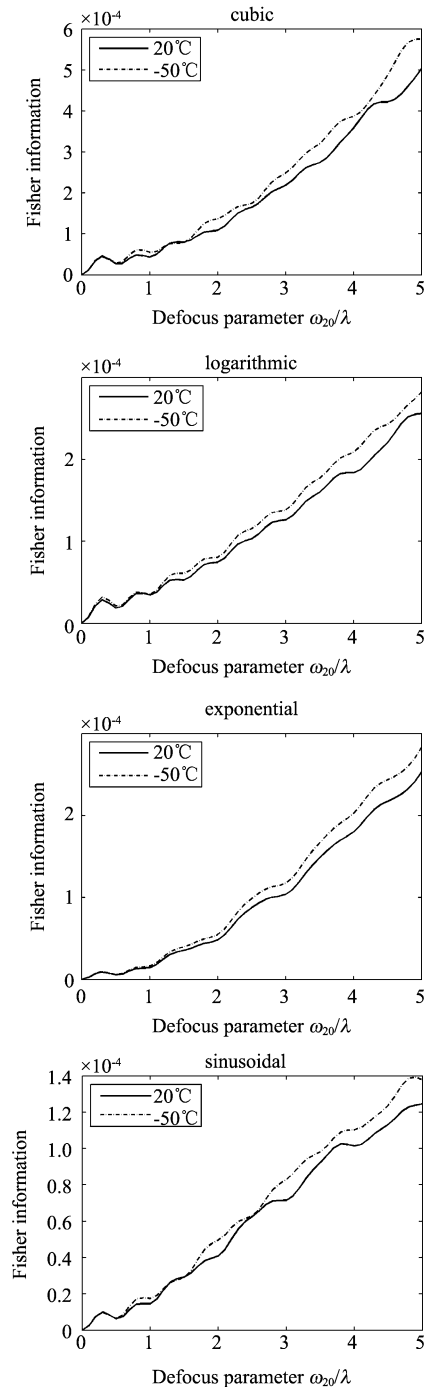


Fig. 2 Comparison of FI curves between original and actual phase masks

图2 考虑相位板热效应前后 FI 随离焦参数变化曲线

modulation to incident light waves in the pupil plane is smaller than the original phase masks when the temperature  $\Delta t$  is negative. Generally, the infrared images include a bigger noise than the images of visible-light waveband. An exact evaluation of the recoverability of wavefront-coded athermalized systems is necessary in the optimization procedure of phase mask

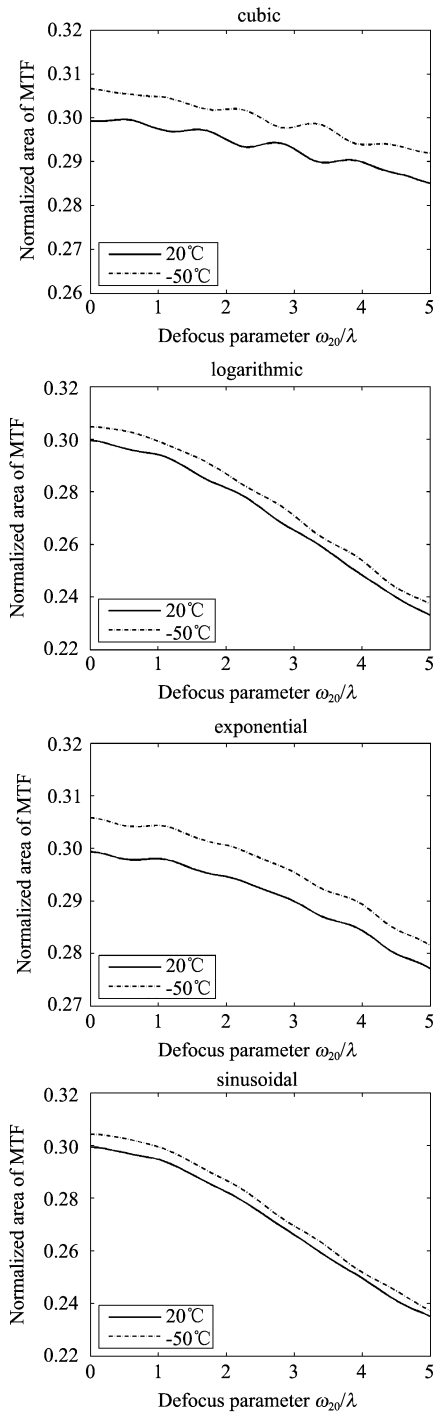


Fig. 3 Comparison of integral areas of MTFs curves between the original and actual phase masks

图3 考虑相位板热效应前后系统 MTF 积分面积的变化

parameters in order to obtain a real signal to noise ratio.

Finally, simulated imaging as a more direct way is used to assess the difference for two types of phase masks (original and actual), and the restored images are shown in Fig. 4. The middle images are the product of simulated images and two dimensional actual

defocused OTFs with thermal effect considered. The filters used are  $H_0/H_{\text{original}}$  and  $H_0/H_{\text{actual}}$  corresponding to the original phase masks and the actual phase masks respectively, where  $H_0$  denotes the incoherent diffraction-limited systems' OTF,  $H_{\text{original}}$  represents the in-focus OTFs for the original phase masks and  $H_{\text{actual}}$  represents the in-focus OTFs for the actual phase masks. All the defocus parameters ( $\omega_{20}$ ) of the intermediate blurred images are set as  $2.5\lambda$ .

Clearly, the original phase masks bring in more artifacts during the restoration process, indicating that the thermal effect on phase mask brings in a considerable change of phase distribution in the pupil plane. The PSNR (Peak Signal-to-Noise Ratio) is a error metric used to compare image restoration quality<sup>[12]</sup>. The higher the PSNR, the better the quality of the restored image is. The PSNRs of the restored images in Fig. 4 are shown in Table 1. As is shown in Table 1, there is an obvious improvement of images' PSNR when the thermal effects on phase masks are considered in the restoration procedure. Therefore, the thermal effect of phase mask should be considered in the optimization procedure in order to ensure the quality of restored images.

Table 1 PSNR of deblurred images with and without thermal effect considered

表1 考虑相位板热效应前后解码图像 PSNR

	Cubic	Logarithmic	Exponential	Sinusoidal
Masks without thermal effect	16.67	19.19	16.52	15.94
Masks with thermal effect	21.75	23.60	24.14	21.87

### 3 Conclusions

In this paper we have discussed the thermal effect on phase mask on the performance of wavefront-coded athermalized imaging systems. We quantified, and demonstrated this thermal effect by the properties of out-of-focus MTFs and simulated images. The results showed that the system taking into account the thermal effect on phase mask had an obvious change in the similarity of out-of-focus MTFs and the recoverability of blurred encoding images. The thermal effect should be considered in the optimization procedure of phase mask parameter in order to achieve a comprehensive and rigorous evaluation of the performance of wavefront-coded athermalized imaging systems. More importantly, we could obtain the accurate PSFs/OTFs at every temperature point to ensure the quality for restored images when the thermal effect

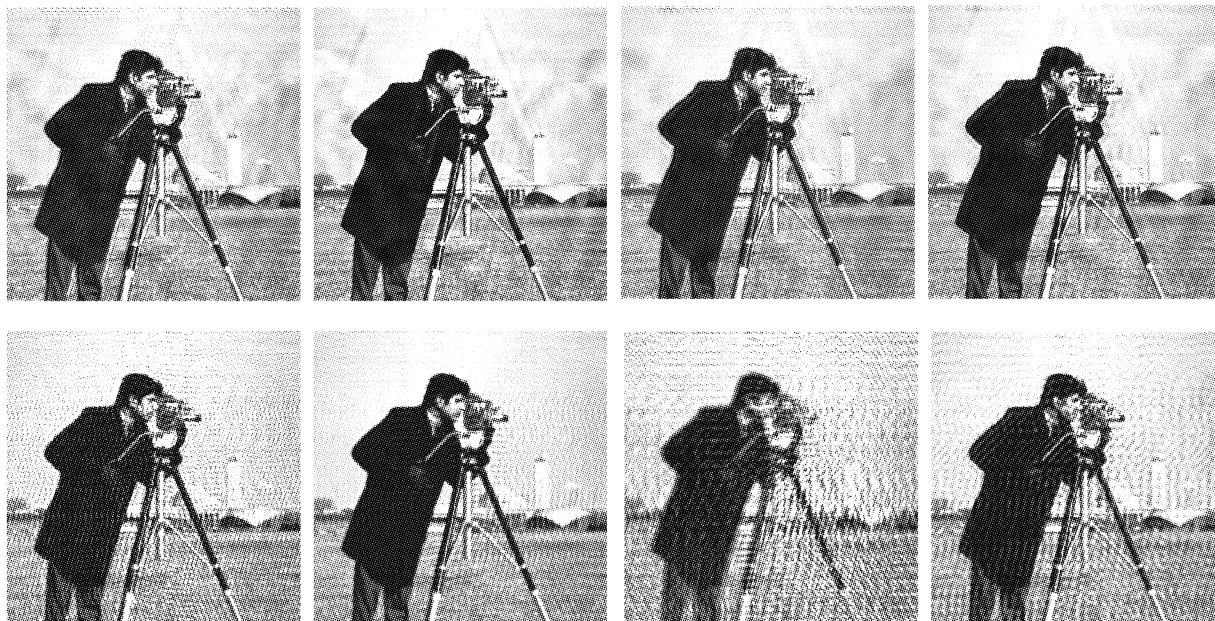


Fig. 4 Inverse filtered images (rows: original filter; row2: actual filter; columns: CPM, LPM, EPM and SPM; all the defocused parameters of encoded images are  $2.5\lambda$ )

图4 复原后图像(1行:原始滤波器,2行:实际滤波器;1列至4列:CPM,LPM,EPM,SPM;全部编码图像的离焦参数为 $2.5\lambda$ )

on phase mask is considered.

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