# Attitude direction estimation of space target parabolic antenna loads using sequential terahertz ISAR images 

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#### Abstract

To monitor the working state of a space target，attitude direction estimation of parabolic antenna loads from a multi－view sequence of the terahertz（ THz ）inverse synthetic aperture radar（ISAR）images is developed． A space－based THz radar imaging system，which aims to achieve surveillance of high earth orbit satellite targets and small satellite targets，is proposed．Under the theorem that the projection of the parabolic antenna edge（a cir－ cle）along arbitrary observation direction is an ellipse，an improved Randomized Hough Transform is proposed to automatically detect and calculate the five key parameters of ellipse components from each THz ISAR image．To ensure the efficiency，accuracy，and robustness of the estimated attitude direction，a two－level estimation algo－ rithm is proposed．The radius and three－dimensional center location of the antenna edge are estimated first．Then， taking these parameters as prior information，the attitude direction is estimated by solving an optimization to mini－ mize the joint error about the length of semi－minor axis and the inclination angle of an ellipse．Electromagnetic scattering data of satellite model targets illustrate the effectiveness and robustness of the proposed method in atti－ tude direction estimation of parabolic antenna loads．


Key words：attitude direction estimation，parabolic antenna load，space target，inverse synthetic aperture radar （ISAR）image，space－based terahertz radar，randomized Hough transform（RHT），particle swarm optimization （PSO）
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## 基于太赫兹 ISAR 图像序列的空间目标抛物面天线载荷指向估计

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#### Abstract

摘要：为了监视空间目标的工作状态，开展了基于多视角太赫兹逆合成孔径雷达（ISAR）图像序列的抛物面天线载荷指向估计研究。提出了一种空基太赫兹雷达成像体制，可以实现高轨卫星目标和小卫星目标的监视。由于圆形的扡物面天线边缘沿着任意观测平面的投影均为梢圆，提出了一种改进的随机霍夫变换方法，可以实现太赫兹ISAR图像中精圆成分的自动检测和参数提取。为了确保指向估计算法的效率，精度和鲁棒性，提出了一种两层估计算法。首先估计扡物面天线边缘的三维圆心坐标和半径，将估计得到的参数作为先验信息，通过求解一个最小化梢圆短轴长度和精圆倾角联合误差的最优化问题实现对天线载荷指向的估计。卫星模型的电磁散射数据证明了所提出的抛物面天线载荷指向估计方法的有效性。 关 键 词：姿态指向估计；把物面天线载荷；空间目标；逆合成孔径雷达（ISAR）图像；空基太赫兹雷达；随机䍜夫変换（RHT）；粒子群优化（PSO） 中图分类号： 045 文献标识码： A


## Introduction

To feed demands of national security and development of science and economy, many space-based systems, such as satellites and space stations, have been launched into space. Parabolic antenna load is a major component of satellite communication system, and its attitude direction and size are important information for monitoring and analyzing the working state and potential intention of a space target.

Until now, the detection sensors of space targets mainly include optical telescopes ${ }^{[1-2]}$ and ground-based radars ${ }^{[3-5]}$. The optical telescopes are capable of observing space targets with high resolution, but the quality of the observation image is heavily dependent on the illumination condition. The ground-based radars can achieve all-day and all-weather monitoring and inverse synthetic aperture radar (ISAR) imaging of space targets and have become the most commonly used means in space target surveillance. However, it still exists some defects. Firstly, limited by the transmitting power, it is hard to detect high earth orbit (HEO) satellite targets and small satellite targets because of the remote transmission distance and small radar cross section (RCS). Secondly, at present, the ground-based radars are all operating below Wband to reduce the impact of atmospheric attenuation, which restricts the imaging resolution. Thus, it cannot achieve fine imaging and identification of space targets at component level. Thirdly, the change of the line of sight (LOS) elevation angle is inadequate to match up with that of the azimuth angle when observing a HEO target, which will lead to an unbalanced distribution of attitude parameters. If the observation diversity of LOS angles is significantly insufficient, it is difficult to estimate the accurate attitude direction of satellite loads, such as parabolic antennas and panels. To overcome these shortcomings, this paper proposes a space-based terahertz ( THz ) radar imaging system, which can achieve high-resolution imaging and identification of key components in both high earth orbit satellite targets and small satellite targets. THz waves are generally referred to the spectrum from 0.1 to 10 THz , which lies in the gap between the microwave and infrared. The space-based THz radar system holds the following characteristics: 1) THz radar is easy to achieve higher carrier frequency and greater usable bandwidth, which enables better spatial resolution and provides more details of space targets ; 2) THz radar is small in size and light in weight. The carrier can flexibly adjust the orbit altitude to achieve sufficient observation of HEO satellites. Until now, THz SAR and ISAR imaging technology have made significant progress in ground and airborne applications ${ }^{[6-10]}$ and showed great advantages in high-resolution imaging and target recognition. Considering that the atmospheric attenuation of THz wave in space is limited, the space-based THz radar holds great potential in space target surveillance.

Reviewing its development, the working state analysis technology of space targets by radar sensors generally can be sorted into two categories. The first works by matching the target features extracted from the radar
echoes or images through maximum likelihood search with a pre-existing database ${ }^{[11-13]}$, such as the RCS, the high-resolution range profiles, and the ISAR images. This sort of data-driven method performs well in some specific missions, but the searching process is too timeconsuming to feed some circumstances with efficiency requirements. Besides, the method would fail in monitoring an unknown non-cooperative target. The second approach is to perform the three-dimensional (3-D) reconstruction. In Ref. [14], multi-static radar is adapted to reconstruct the 3-D geometry of low earth orbit (LEO) space targets through interference technology. However, this method depends on the complex radar system structure, and has not been widely used. At present, the most commonly used 3-D reconstruction approach is based on the sequential ISAR images ${ }^{[15-20]}$. In these literatures, the singular value decomposition (SVD) method ${ }^{[15-16]}$ and factorization method ${ }^{[17-20]}$ utilized in optical image reconstruction ${ }^{[21-22]}$ were applied to decompose the multi-perspective range-Doppler (RD) history projection matrix into the LOS angle unit vector matrix and the target 3-D reconstruction result. Before 3-D reconstruction, the association of scattering points from different ISAR images should be accomplished in these methods. The commonly used scattering trajectory association methods include Kalman filter method ${ }^{[23]}$, Kanade - Lucas - Tomasi (KLT) feature tracker method ${ }^{[19]}$, and scale invariant feature transform (SIFT) method ${ }^{[20]}$. It should be noted that all these scattering trajectory association methods are based on the precarious point scattering center model assumption, which omit the angular glint phenomenon ${ }^{[24]}$ in high-frequency radar measurements. In the THz band, Liu et al. ${ }^{[25]}$ have investigated the aspect-dependent scattering characteristics of complex targets at 0.34 THz , and the experimental results showed that the scattering persistence angles of aspect-dependent scattering centers are no more than $5.5^{\circ}$. Thus, the complex structure of space targets will lead to a huge decline in precision and efficiency of scattering trajectory association from sequential ISAR images, which increases the failure risk of the 3-D reconstruction. As a result, it is difficult to achieve accurate attitude estimation in practical applications. To overcome the defects of point scattering center model-based 3-D reconstruction methods, Zhou et al. ${ }^{[26]}$ have explored the shape feature of LEO satellite targets within the ground-based Ku-band ISAR sequence and achieved attitude estimation of rectangular components such as solar panels. For HEO satellite targets and small satellite targets that are mainly responsible for communication tasks, the size and attitude direction estimation of parabolic antenna loads is necessary for monitoring and analyzing their working state and potential intention. However, there is almost no relevant research at present.

In this paper, we propose a complete set of algorithms to estimate the attitude direction of parabolic antenna loads from sequential THz ISAR images, including sequential ISAR images acquisition, parabolic antenna components detection, and attitude direction estimation.

The algorithms explore continuous changing of circular structures among sequential ISAR images to interpret the attitude direction information．The electromagnetic scat－ tering characteristic of parabolic antenna is investigated． To obtain the significant edge information of parabolic an－ tenna loads，the cross－polarization echo data of a satellite model are calculated．Under the theorem that the projec－ tion of the edge of a parabolic antenna（a circle）on arbi－ trary two－dimensional（2－D）plane is an ellipse，an im－ proved Randomized Hough Transform（RHT）is pro－ posed to automatically detect and calculate the five key parameters including center coordinates，lengths of semi－ major axis and semi－minor axis，and inclination angle of ellipses from each THz ISAR image．The attitude direc－ tion is estimated based on a two－level estimation algo－ rithm．Firstly，the antenna radius is estimated by averag－ ing the lengths of semi－major axis，and the 3－D center lo－ cation of the antenna edge is obtained through least squares estimation based on the detected center coordi－ nates of ellipses and LOS angle projection matrix．Final－ ly，taking the antenna radius and 3－D center location as prior information，the attitude direction is estimated through solving an optimization to minimize the joint er－ ror about the length of semi－minor axis and the inclina－ tion angle based on the particle swarm optimization （PSO）algorithm ${ }^{[27]}$ ．

This paper is organized as follows．In Sect．1，The observation geometry of space－based THz radar imaging system is described in detail．The electromagnetic scat－ tering characteristic of parabolic antenna is investigated in Sect．2．In Sect．3，the implementation details of the proposed attitude direction estimation method are provid－ ed．Sect． 4 is the experimental part，including imaging performance comparison，attitude direction parameter es－ timation，and error analysis．Finally，conclusions are drawn in Sect． 5.

## 1 Observation geometry of space－based terahertz radar imaging system

The space target observation geometry of space－ based terahertz radar is illustrated in Fig．1，in which $O$－ $X Y Z$ represents the earth centered inertial（EIC）coordi－ nate system，$O-X_{r} Y_{r} Z_{r}$ represents the measurement coor－ dinate system of space－based terahertz radar，and $O$－ $X_{t} Y_{t} Z_{t}$ represents the target coordinate system，respec－


Fig． 1 Space target observation geometry of space－based tera－ hertz radar．
图1 空基太赫兹雷达空间目标观测几何
tively．For a three－axis stabilized space target，its atti－ tude will keep unchanged relative to the target coordinate system，and the projection on ISAR image plane depends on the instantaneous radar LOS angles $\theta_{\mathrm{r}}$ and $\varphi_{\mathrm{r}}$ ，which can be obtained from the ground－based radar tracking sys－ tem ${ }^{[28]}$ or the developed space－based terahertz radar tracking system．Supposing that the 3－D unit vector of at－ titude direction of a parabolic antenna load is

$$
\begin{equation*}
\hat{\boldsymbol{n}}_{\mathrm{t}}=(\cos \alpha \sin \beta, \cos \alpha \cos \beta, \sin \alpha)^{\mathrm{T}} \tag{1}
\end{equation*}
$$

where $\alpha$ and $\beta$ denote the elevation angle and azimuth an－ gle of attitude direction，and they are defined in the same way as radar LOS angles $\theta_{\mathrm{r}}$ and $\varphi_{\mathrm{r}}$ ．

Based on the coordinate system transformation，the 3－D unit vector of the attitude direction in the radar mea－ surement coordinate system can be expressed as

$$
\begin{equation*}
\hat{n}_{\mathrm{r}}=T_{\mathrm{er}} \cdot \boldsymbol{T}_{\mathrm{te}} \cdot \hat{n}_{\mathrm{t}} \tag{2}
\end{equation*}
$$

where $\boldsymbol{T}_{t e}$ is the transformation matrix from target coordi－ nate system to EIC coordinate system，and $\boldsymbol{T}_{\text {er }}$ is the transformation matrix from EIC coordinate system to ra－ dar measurement coordinate system． $\boldsymbol{T}_{\mathrm{te}}$ and $\boldsymbol{T}_{\text {er }}$ are ex－ pressed as

$$
\begin{gather*}
\boldsymbol{T}_{\mathrm{te}}=\left[\begin{array}{ccc}
\cos \varphi_{\mathrm{te}} & -\sin \varphi_{\mathrm{te}} & 0 \\
\cos \theta_{\mathrm{te}} \sin \varphi_{\mathrm{te}} & \cos \theta_{\mathrm{te}} \cos \varphi_{\mathrm{te}} & -\sin \theta_{\mathrm{te}} \\
\sin \theta_{\mathrm{te}} \sin \varphi_{\mathrm{te}} & \sin \theta_{\mathrm{te}} \cos \varphi_{\mathrm{te}} & \cos \theta_{\mathrm{te}}
\end{array}\right],  \tag{3}\\
\boldsymbol{T}_{\mathrm{er}}=\left[\begin{array}{ccc}
\cos \varphi_{\mathrm{er}} & -\sin \varphi_{\mathrm{er}} & 0 \\
\cos \theta_{\mathrm{er}} \sin \varphi_{\mathrm{er}} & \cos \theta_{\mathrm{er}} \cos \varphi_{\mathrm{er}} & -\sin \theta_{\mathrm{er}} \\
\sin \theta_{\mathrm{er}} \sin \varphi_{\mathrm{er}} & \sin \theta_{\mathrm{er}} \cos \varphi_{\mathrm{er}} & \cos \theta_{\mathrm{er}}
\end{array}\right] . \tag{4}
\end{gather*}
$$

Similarly，for a scattering point located on the space target with 3－D coordinates $\boldsymbol{L}_{\mathrm{t}}=\left(x_{\mathrm{t}}, y_{\mathrm{t}}, z_{\mathrm{t}}\right)^{\mathrm{T}}$ ，the 3－D coor－ dinates in radar measurement coordinate system can be transformed as

$$
\begin{equation*}
\boldsymbol{L}_{\mathrm{r}}=\boldsymbol{T}_{\mathrm{er}} \cdot \boldsymbol{T}_{\mathrm{te}} \cdot\left(x_{\mathrm{t}}, y_{\mathrm{t}}, z_{\mathrm{t}}\right)^{\mathrm{T}} \tag{5}
\end{equation*}
$$

According to the ISAR imaging theory，the imaging process of space targets is to project 3－D scattering points on a 2－D plane based on the radar measurement LOS an－ gles．The projected 2－D coordinates on an ISAR image can be expressed as
$\left[\begin{array}{l}y_{\mathrm{r}, \mathrm{f}} \\ x_{\mathrm{r}, \mathrm{f}}\end{array}\right]=\boldsymbol{O}_{\mathrm{f}} \cdot \boldsymbol{L}_{\mathrm{r}}=\left[\begin{array}{ccl}\cos \theta_{\mathrm{r}, \mathrm{f}} \sin \varphi_{\mathrm{r}, \mathrm{f}} & \cos \theta_{\mathrm{r}, \mathrm{f}} \cos \varphi_{\mathrm{r}, \mathrm{f}} \sin \theta_{\mathrm{r}, \mathrm{f}} \\ \cos \varphi_{\mathrm{r}, \mathrm{f}} & -\sin \varphi_{\mathrm{r}, \mathrm{f}} & 0\end{array}\right] \cdot \boldsymbol{L}_{\mathrm{r}}$
where $\boldsymbol{O}$ denotes the LOS angle projection matrix，and subscript $f$ denotes the serial number of ISAR images．It should be noted that the projected 2－D coordinates are de－ rived under the circumstance that the azimuth angle $\varphi_{\mathrm{r}}$ changes continuously，and the elevation angle $\theta_{\mathrm{r}}$ is al－ most constant within one ISAR imaging period．

Compared with traditional ground－based radars，a significant advantage of the space－based terahertz radar system is that the LOS elevation angle $\theta_{\mathrm{r}}$ can be adjust flexibly through orbital transfer of the carrier，which helps to achieve sufficient observation of space targets with almost unchanged orbit inclination，such as HEO satellites and geosynchronous satellites．Sufficient LOS observation angles will ensure the accuracy and robust－
ness of the attitude direction estimation algorithm．
This paper focuses on the attitude direction estima－ tion of parabolic antenna loads，so the mature ISAR im－ aging theories are not described in detail．In the last few decades，the ISAR imaging methods have been intensive－ ly investigated，and representative works can be found in ${ }^{[29-31]}$ ．It should be noted that the terahertz radar holds a very high range resolution，and the range cell migration correction（RCMC）${ }^{[32]}$ must be performed in the tera－ hertz ISAR imaging process．

## 2 Electromagnetic scattering character－ istic of parabolic antenna



Fig． 2 Geometry model of a paraboloid．
图2 抛物面几何模型

The geometry model of a paraboloid under Cartesian coordinate system is shown as Fig．2，and its Cartesian coordinate equation is

$$
\begin{equation*}
y=\frac{x^{2}+z^{2}}{4 F} \tag{7}
\end{equation*}
$$

where $F$ represents the focal length of the paraboloid． Considering that the paraboloid is rotationally symmet－ ric，its projection，which is a parabola，on $x-y$ plane is investigated．In Fig．2，$D$ denotes the diameter，$\varphi$ de－ notes the incident angle of plane wave transmitted by the radar，$G$ denotes the focal point，$P$ denotes the specular reflection point on the parabola with respect to the inci－ dent plane wave，$\rho$ denotes the length from $G$ to $P$ ，and $\eta$ denotes the included angle between the $y$－axis and the line segment $F P$ ．A widely known conclusion about the parabola is $\eta=2 \varphi$ ，then the polar coordinate equation of the parabola can be expressed as

$$
\begin{equation*}
\rho=\frac{2 F}{1+\cos (2 \varphi)} \tag{8}
\end{equation*}
$$

It can be seen from Fig． 2 that the position of the specular reflection point is sliding along the parabola with the continuous change of the incident angle of plane wave，and the corresponding angle range can be calculat－ ed as

$$
\begin{equation*}
\varphi \in\left[-\frac{1}{2} \arctan \left(\frac{8 D F}{16 F^{2}-D^{2}}\right), \frac{1}{2} \arctan \left(\frac{8 D F}{16 F^{2}-D^{2}}\right)\right] \tag{9}
\end{equation*}
$$

Due to this special characteristic，the paraboloid be－ longs to the sliding scattering center ${ }^{[33]}$ ，and the specular reflection point makes a major contribution to the RCS． Apart from the specular reflection point，the edge of the paraboloid will also contribute to the RCS ${ }^{[34]}$ ．However， once the specular reflection point exists，this minor con－ tribution is hard to be noticed．

Based on the geometry optics theory ${ }^{[35]}$ ，the sliding scattering center model，whose phase reference center is the bottom center of the paraboloid，of the specular re－ flection point can be established as

$$
\begin{align*}
\left|\sqrt{\sigma_{G O}(k, \varphi)}\right| & =\sqrt{\pi\left|\rho_{1} \rho_{2}\right|} \exp (-j 2 k F \sin \varphi \tan \varphi) \\
& =\frac{2 \sqrt{\pi} F}{\cos ^{2} \varphi} \exp (-j 2 k F \sin \varphi \tan \varphi) \tag{10}
\end{align*}
$$

where $k=2 \pi f_{r} / c$ denotes the wavenumber，$c$ denotes the speed of light，and $f_{r}$ denotes the operating frequency of radar．$\rho_{1}$ and $\rho_{2}$ are the two－principal radius of the curved surface at specular reflection point ${ }^{[36]}$ ，and they are expressed as

$$
\begin{equation*}
\rho_{1}=-\frac{2 F}{\cos \varphi}, \rho_{2}=-\frac{2 F}{\cos ^{3} \varphi} \tag{11}
\end{equation*}
$$

Due to the position of the specular scattering point changes with the incident angle of plane wave，it is im－ possible to achieve association of any fixed scattering point on the surface of a parabolic antenna from different ISAR images，let alone reconstruct the 3－D geometry． Fortunately，the cross－polarization radar echoes provide a solution to this problem．Wang et al．${ }^{[37]}$ have illustrat－ ed that the cross－polarization radar image is more sensi－ tive to the edges and corners．In this paper，the variant shape feature of the parabolic antenna edge among the cross polarized ISAR image sequence is adapt to inter－ pret the attitude direction information．Utilizing this sort of high－level image feature for parameter estimation avoids the association difficulty in conventional factoriza－ tion method．

To verify the reliability of the theory illustrated in this section，electromagnetic simulations of a parabolic anten－ na model are performed．The diameter and focal length of the paraboloid are 0.48 m and 0.24 m ，respectively． From Eq．9，we can calculate that the range of incident angle $\varphi$ with specular reflection point is $\left[-26.565^{\circ}\right.$ ， 26． $565^{\circ}$ ］．The material is perfect electric conductor， and the geometric position is the same as Fig．2．To sim－ ulate the real parabolic antenna load of space targets，the thickness of the antenna edge is set to 1 cm ．The electro－ magnetic calculations are based on the physical optics， and the edge scattering has been taken into consider－ ation ${ }^{[38-39]}$ ．The operating frequency of the THz radar rang－ es from 0.215 THz to 0.225 THz with 51 uniform sample points．

In the first simulation，the parabolic antenna model
is static，and the incident angle $\varphi$ ranges from $-90^{\circ}$ to $90^{\circ}$ with $0.2^{\circ}$ sample interval．The RCS curve at 0.22 THz is shown in Fig．3．From Fig．3，we can conclude three conclusions：1）The scattering center model estab－ lished in Eq．1）well matches the electromagnetic calcu－ lation result，and the undulation of the RCS curve is mainly caused by the coherent accumulation of the scat－ tering component contributed by antenna edge ；2）When the LOS angle is not perpendicular to the antenna edge， the scattering intensity of specular reflection point is far more than that of antenna edge；3）Compared with the specular reflection point，the antenna edge has a depolar－ ization effect，and the scattering intensity of cross polar－ ization has a small diversity with respect to the incident angle．


Fig． 3 RCS of the parabolic antenna model at 0.22 THz ．图3 0.22 THz 抛物面天线模型 RCS

In the second simulation，the parabolic antenna model rotates $3^{\circ}$ around the $y$－axis with $0.05^{\circ}$ sample in－ terval，and the incident angle $\varphi$ is set $25^{\circ}$ and $60^{\circ}$ ，re－ spectively．Figure 4 shows the ISAR imaging results． Comparing Fig． 4 （a）with Fig．4（b），we can find that when the specular reflection point exists，the information of the other part in the parabolic antenna are greatly sup－ pressed，and it is hard to judge whether a parabolic an－ tenna exists or not．Comparing Fig． 4 （b）with Fig． 4 （c），it is obvious that the VV polarization image is sensi－ tive to the face feature，but the VH polarization image is sensitive to the edge feature．Thus，the cross polarized ISAR images are more suitable for the attitude direction estimation of parabolic antenna loads．

## 3 Attitude direction estimation algorithm

## 3． 1 Overall Algorithm Framework

Based on the basic theories in Section II and Section III，the complete attitude direction estimation algorithm of parabolic antenna loads is introduced in this section． The overall process of the attitude direction estimation scheme is concluded as the following steps：

Step 1：Adjust the orbit parameters of the space－ based THz radar system to achieve a sufficient and effec－ tive observation of the parabolic antenna loads on a space target．The effective observation means that the LOS an－ gle should be far away from the axis of parabolic antenna．

Step 2：Transform the LOS angle parameters ob－ tained by the radar tracking system under EIC coordinate system into these under the radar measurement coordi－ nate system based on the geometric relations described in Eq． 2.

Step 3：Adopt the RD algorithm with RCMC to ob－ tain the sequential high－resolution THz ISAR images of the observed space target from the cross－polarized radar echoes．The azimuth scaling is achieved based on the LOS angle parameters and the sampling interval of slow time．If the ISAR images feed the requirement of attitude direction estimation，go to Step 4，otherwise，go to Step 1.

Step 4：The ISAR images are denoised and grayed． Utilize the morphology methods to corrode the target out－ line in each ISAR image，and the typical rectangle com－ ponents can be removed based on the Radon transforma－ tion ${ }^{[40]}$ or K－means clustering algorithm to reduce the computation complexity in the following parabolic anten－ na component detection operation．

Step 5：Perform the proposed improved RHT on each ISAR image to extract the five key parameters， which include the 2－D center coordinates，the length of semi－major axis and semi－minor axis，and the inclination angle，of the ellipse components projected by the anten－ na edge．

Step 6：Build the geometric projection matrix of each ISAR image with respect to the radar LOS angles， and estimate the antenna radius and the center coordi－ nates of the antenna edge based on the extracted ellipse parameters in Step 5.

Step 7：Take the antenna radius and center location


Fig． 4 ISAR images of the parabolic antenna $\operatorname{model}(\mathrm{a}) \varphi=25^{\circ}$ ，VV polarization；（b）$\varphi=60^{\circ}, \mathrm{VV}$ polarization；（c）$\varphi=60^{\circ}$ ，VH polariza－ tion
图4 抛物面天线模型 $\operatorname{ISAR}$ 图像（a）$\varphi=25^{\circ}$ ，VV极化；（b）$\varphi=60^{\circ}$ ，VV极化；（c）$\varphi=60^{\circ}$ ，VH 极化
as prior information，and estimate the attitude direction through an optimization to minimize the joint error about the length of semi－minor axis and the inclination angle．

To show the process clearly，a flowchart is given in Fig．5．The details of the key steps are given in the fol－ lowing subsections．


Fig． 5 Flowchart of the attitude direction estimation process图5 姿态指向估计流程图

## 3． 2 Projection of parabolic antenna edge

An innovation of this paper is utilizing the robust shape feature of the parabolic antenna edge to replace the invalid point scattering center feature in the attitude di－ rection estimation application．The edge of a parabolic antenna can be seen as a circle in 3－D free space，and ex－ cept the extreme condition that the LOS angle is perpen－ dicular to the circle，its projection along arbitrary LOS angle direction is an ellipse．Based on this theorem，the connection between the sequential ISAR images and structural characteristic of parabolic antenna loads can be established．

In the 3－D free space，a circle cannot be described by an explicit Cartesian coordinate equation，but it can be uniquely determined by the 3－D center coordinates， radius，and normal vector．For the parabolic antenna load，the normal vector corresponds to the attitude direc－ tion．Suppose the 3－D center coordinates，radius，and at－ titude angles of the edge of a parabolic antenna in the measurement coordinate system are $\left(x_{\mathrm{a}}, y_{\mathrm{a}}, z_{\mathrm{a}}\right), r_{a}$ ，and $\left(\alpha_{\mathrm{a}}, \beta_{\mathrm{a}}\right)$ ，respectively．The parametric equations of the antenna edge can be expressed as

$$
\left\{\begin{array}{l}
x(\varepsilon)=x_{\mathrm{a}}+r_{\mathrm{a}} \cos (\varepsilon) u_{1}+r_{\mathrm{a}} \sin (\varepsilon) v_{1}  \tag{12}\\
y(\varepsilon)=y_{\mathrm{a}}+r_{\mathrm{a}} \cos (\varepsilon) u_{2}+r_{\mathrm{a}} \sin (\varepsilon) v_{2} \\
z(\varepsilon)=z_{\mathrm{a}}+r_{\mathrm{a}} \cos (\varepsilon) u_{3}+r_{\mathrm{a}} \sin (\varepsilon) v_{3}
\end{array}\right.
$$

where $\left(u_{1}, u_{2}, u_{3}\right)$ and（ $v_{1}, v_{2}, v_{3}$ ）denote two unit vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ ，respectively． $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular to the nor－ mal vector $\hat{\boldsymbol{n}}_{r}$ in Eq．2，and they are also perpendicular to each other．$\varepsilon$ is the rotation angle，which ranges from 0 to $2 \pi$ ．Then，the projected 2－D coordinates $\left(x_{\mathrm{r}}, y_{\mathrm{r}}\right)$ of the circle on each ISAR image can be obtained from

Eq． 6 based on the LOS angle data．The corresponding projected ellipse on a 2－D plane can be expressed by

$$
\begin{gather*}
\mu_{1} x_{\mathrm{r}}^{2}+\mu_{2} x_{\mathrm{r}} y_{\mathrm{r}}+\mu_{3} y_{\mathrm{r}}^{2}+\mu_{4} x_{\mathrm{r}}+\mu_{5} y_{\mathrm{r}}+1=0  \tag{13}\\
\text { s.t. } \mu_{2}^{2}-4 \mu_{1} \mu_{3}<0
\end{gather*}
$$

where $\overrightarrow{\boldsymbol{\mu}}=\left\{\mu_{1}, \mu_{2}, \mu_{3}, \mu_{4}, \mu_{5}\right\}$ denotes the parameter set of an ellipse，and it can be precisely calculated by solv－ ing a linear equation with more than four coordinate points．In addition to this general expression，an ellipse can also be determined by five key parameters including the 2－D center coordinates，the lengths of semi－major ax－ is and semi－minor axis，and the inclination angle．In this case，the ellipse can be expressed as

$$
\begin{align*}
& \frac{\left[\left(x_{\mathrm{r}}-x_{\mathrm{e}}\right) \sin (\gamma)+\left(y_{\mathrm{r}}-y_{\mathrm{e}}\right) \cos (\gamma)\right]^{2}}{a^{2}} \\
+ & \frac{\left[-\left(x_{\mathrm{r}}-x_{\mathrm{e}}\right) \cos (\gamma)+\left(y_{\mathrm{r}}-y_{\mathrm{e}}\right) \sin (\gamma)\right]^{2}}{b^{2}}=1, \tag{14}
\end{align*}
$$

where $x_{\mathrm{e}}$ and $y_{\mathrm{e}}$ denote the 2－D center coordinates，$a$ and $b$ denote the lengths of two semi－axis，and $\gamma$ denotes the inclination angle．The relations between the five key pa－ rameters and the parameter set $\overrightarrow{\boldsymbol{\mu}}$ are

$$
\begin{gather*}
x_{\mathrm{e}}=\frac{\mu_{2} \mu_{5}-2 \mu_{3} \mu_{4}}{4 \mu_{1} \mu_{3}-\mu_{2}^{2}}  \tag{15}\\
y_{\mathrm{e}}=\frac{\mu_{2} \mu_{4}-2 \mu_{1} \mu_{5}}{4 \mu_{1} \mu_{3}-\mu_{2}^{2}}  \tag{16}\\
b=\sqrt{\frac{2\left(\mu_{1} x_{\mathrm{e}}^{2}+\mu_{3} y_{\mathrm{e}}^{2}+\mu_{2} x_{\mathrm{e}} y_{\mathrm{e}}-1\right)}{\mu_{1}+\mu_{3}+\sqrt{\left(\mu_{1}-\mu_{3}\right)^{2}+\mu_{2}^{2}}}}  \tag{17}\\
\gamma=\sqrt{\frac{2\left(\mu_{1} x_{\mathrm{e}}^{2}+\mu_{3} y_{\mathrm{e}}^{2}+\mu_{2} x_{\mathrm{e}} y_{\mathrm{e}}-1\right)}{\mu_{1}+\mu_{3}-\sqrt{\left(\mu_{1}-\mu_{3}\right)^{2}+\mu_{2}^{2}}}}  \tag{18}\\
\gamma=0.5 \arctan \left(\frac{\mu_{2}}{\mu_{1}-\mu_{3}}\right) \tag{19}
\end{gather*}
$$

From Eq． 12 to Eq．19，the complete projection re－ lation from a 3－D circle to a 2－D ellipse is established． Once the parameters of a circle and LOS angle data are known，the five key parameters of the projected ellipse can be directly obtained．In this subsection，the deriva－ tion of complex geometric relations is avoided by solving a simple linear equation．

## 3． 3 Automatic ellipse components extraction

Detecting specific curves（straight line，circle，el－ lipse，etc．）from an optical image is one of the basic tasks in computer vision，and the commonly used curve detecting methods are the Hough Transform（HT）and its variants ${ }^{[41]}$ ．These methods transform one pixel of the im－ age space into a parameterized curve of the parameter space，and each parameter coordinate is used to repre－ sent a curve segment in the image space．However，the algorithm complexity increases exponentially with the in－ crease of the number of parameters，which will result in a very long computing time if the number of parameters is larger than three．To overcome the shortcomings of HT， the Random Hough Transform（RHT）and its vari－ ants ${ }^{[42-43]}$ ，which explore the shape features of the detect－
ed curves，have been investigated to extract the parame－ ters of specific curves．Compared with HT，the RHT holds the advantages in high accuracy，low storage，low computing complexity，and infinite parameter space．Un－ til now，the RHT has achieved big success in curve de－ tection from optical images．

In this paper，we improve the RHT to achieve el－ lipse detection from the ISAR images．Different from the optical images，the projected ellipse edge of the parabol－ ic antenna load on ISAR images has a certain thickness because of the limited resolution of radars，which can be seen from Fig．4（c）．To guarantee the precision and ro－ bustness in ellipse detection，the improved RHT utilizes the five or more strongest scattering points of the antenna edge projection to estimate the five key parameters of an ellipse．

We give the improved RHT procedure as follows ：
ISAR image preprocessing：The ISAR images are denoised by the CLEAN algorithm ${ }^{[53]}$ and transformed in－ to binary images．To promote the efficiency of the im－ proved RHT in ellipse detection，some invalid scattering points，such as the target side lobes in RD imaging，are reduced through image corrosion processing，and rectan－ gle components，such as the polar panel，are removed through the Radon transformation or K－means cluster al－ gorithm．

Step 1：Scan a binary image and put the coordinates $p_{i}=\left(x_{i}, y_{i}\right)$ of all the＇on＇pixels into the pixel data set $P$ ．Then，initialize a parameter data set $S=$ null and $k=$ 1.

Step 2：Randomly pick five points $p_{1}, \ldots, p_{5}$ out of $P$ in such a way that all points of $P$ have an equal probabili－ ty to be taken as $p_{1}$ ，then all points of $P-\left\{p_{1}\right\}$ have an equal probability to be taken as $p_{2}$ ，etc．

Step 3：Solve five joint equations of Eq． 13 as $f\left(\mu_{1}, \mu_{2}, \mu_{3}, \mu_{4}, \mu_{5}, x_{i}, y_{i}\right)=0, i=1,2, \ldots, 5$ to determine a parameter point $\left\{\mu_{1}, \mu_{2}, \mu_{3}, \mu_{4}, \mu_{5}\right\}$ ．If $\mu_{2}^{2}-4 \mu_{1} \mu_{3}<0$ ， go to Step 4，otherwise go to Step 1.

Step 4：Calculate a parameter point $s_{k}=$ $\left\{x_{e, k}, y_{e, k}, a_{k}, b_{k}, \gamma_{k}\right\}$ based on Eq．1）to Eq．19．Search among set $S$ for an element $s$ such that $\left\|s_{k}(1: 2)-s(1: 2)\right\|_{2}<\delta_{1},\left|s_{k}(3)-s(3)\right|<\delta_{2}, \mid s_{k}(4)-$ $s(4) \mid<\delta_{3}$ ，and $\left|s_{k}(5)-s(5)\right|<\delta_{4}$ ，where $\delta_{1}, \delta_{2}, \delta_{3}$ ，and $\delta_{4}$ are given tolerances．If found，go to Step 6，other－ wise，go to Step 5.

Step 5：Attach to $s_{k}$ an accumulating cell with score one and insert it into set $S$ as a new element．Go to Step 7.

Step 6：Increase the score of the accumulating cell of $s$ by one，and then check whether the increased score is smaller than a given threshold $n_{p}$（e．g．，$n_{p}=3$ ）．If yes，go to Step 7 ，otherwise，go to Step 8.

Step 7：$k=k+1$ ．If $k>k_{\text {max }}$（e．g．，$k_{\text {max }}=$ 50000 ），then stop，otherwise，go to Step 2.

Step 8：Take $s$ as the parameters of a possible el－ lipse and take out of $P$ all the pixels lying on the curve． If there are $m_{p}$ such pixels and $m_{p}>m_{\text {min }}$ ，then go to Step 9 ；otherwise，$s$ represents a false curve，return the $m_{p}$
pixels into set $P$ ，then take $s$ and its accumulating cell out of set $S$ ，and go to Step 2 ．

Step 9：Keep $x_{e}, y_{e}$ ，and $\gamma$ in $s$ unchanged，and give both $a$ and $b$ a varying range $[-d r, d r$ ］（e．g．，$d r$ is half the range resolution of the THz radar）to generate a mask area．Select the coordinates of the top five or more strongest scattering points corresponding to the mask area in the ISAR image to calculate a parameter point $\hat{s}=$ $\left\{\hat{x}_{e}, \hat{y}_{e}, \hat{a}, \hat{b}, \hat{\gamma}\right\}$ as the final parameters of the detected el－ lipse．It should be noted that the strong scattering points are extracted in such a way that the strongest scattering point is extracted first，and the scattering points within the region of a given radius $l_{s}$ around this strongest scat－ tering point are removed．Then，the next strongest scat－ tering point is extracted，etc．Delete all the pixels corre－ sponding to the mask area in the ISAR image from $P$ ，re－ set $S=$ null and $k=1$ ，and go to Step 2 to detect the next ellipse in the ISAR image．

Perform the improved RHT procedure on the se－ quential THz ISAR images，the successive projected el－ lipse parameters of the parabolic antenna loads can be ob－ tained．Due to the positions of the strongest scattering points utilized to calculate ellipse parameters nearly lo－ cate on the antenna edge，the improved RHT ensures both the accuracy and robustness in the automatic ellipse components extraction process．

## 3． 4 Attitude direction estimation of parabolic an－ tenna loads

In this subsection，we build a series of LOS angle projection matrices to estimate the attitude direction from the automatically extracted ellipse components in the pre－ vious subsection．An intuitive way to estimate the atti－ tude direction is searching the six parameters of the an－ tenna edge at the same time to minimize the difference between the projected ellipse parameters and the detect－ ed ellipse parameters from sequential ISAR images． Based on the structural constraint of parabolic antenna components，the minimization is described as follows：

$$
\begin{align*}
& \min _{x_{a}, y_{a} z_{a} r_{a}, \alpha_{a} \beta_{a}} \sum_{f=1}^{F}\left\{\left\|\left(x_{r e f}, y_{r e f}\right)^{\mathrm{T}}-\left(\hat{x}_{e, f}, \hat{y}_{e, f}\right)^{\mathrm{T}}\right\|_{2}\right. \\
& \left.\quad+\sigma_{1}\left|a_{r, f}-\hat{a}_{f}\right|+\sigma_{2}\left|b_{r, f}-\hat{b}_{f}\right|+\sigma_{3}\left|\gamma_{r, f}-\hat{\gamma}_{f}\right|\right\}, \tag{20}
\end{align*}
$$

where $\left\{x_{r e}, y_{r e}, a_{r}, b_{r}, \gamma_{r}\right\}$ is the ellipse parameter set pro－ jected by the antenna edge，whose parameter set is $\left\{x_{a}, y_{a}, z_{a}, r_{a}, \alpha_{a}, \beta_{a}\right\}$ ，and it can be obtained through Eqs．12－19．$F$ denotes the number of ISAR images，and $\sigma_{1}, \sigma_{2}$ ，and $\sigma_{3}$ denote the weight factors，which balance the confidence of different feature parameters．This pro－ cess requires simultaneous optimization of six parame－ ters，and except $\alpha_{a}$ and $\beta_{a}$ ，the ranges of the other four parameters are unknown．Besides，the optimization of multiple parameters usually takes a long processing time and is easy to fall into a local optimal solution．

To overcome the problems in multi－parameter opti－ mization，a two－level estimation method is proposed in this paper．When a 3－D circle projects to a 2－D ellipse， there are two special characteristics．Firstly，the center of the 2－D ellipse is the projection of the center of the 3－

D circle. Secondly, the length of the semi-major axis of the ellipse is the same as the length of the radius of the circle. Thus, the 3-D center coordinates and radius of the antenna edge can be estimated first. The estimated radius is

$$
\begin{equation*}
\hat{r}_{a}=\frac{1}{F} \sum_{f=1}^{F} \max \left(\hat{a}_{f}, \hat{b}_{f}\right) \tag{21}
\end{equation*}
$$

where max ( ) means to take the maximum value. By collecting the LOS angle projection matrices $\boldsymbol{O}_{f}$ in Eq. 6, the 3-D center coordinates can be estimated through a least squares estimation:
$\left(\hat{x}_{a}, \hat{y}_{a}, \hat{z}_{a}\right)^{\mathrm{T}}=\left(\boldsymbol{O}^{\mathrm{T}} \cdot \boldsymbol{O}\right)^{-1} \cdot \boldsymbol{O}^{\mathrm{T}} \cdot\left(x_{c, 1}, \ldots, x_{c, f}, y_{c, 1}, \ldots, y_{c, f}\right)^{\mathrm{T}}$,
where

$$
\boldsymbol{O}=\left[\begin{array}{ccc}
\cos \theta_{r, 1} \sin \varphi_{r, 1} & \cos \theta_{r, 1} \cos \varphi_{r, 1} & \sin \theta_{r, 1}  \tag{22}\\
\vdots & \vdots & \vdots \\
\cos \theta_{r, f} \sin \varphi_{r, f} & \cos \theta_{r, f} \cos \varphi_{r, f} & \sin \theta_{r, f} \\
\cos \varphi_{r, 1} & -\sin \varphi_{r, 1} & 0 \\
\vdots & \vdots & \vdots \\
\cos \varphi_{r, f} & -\sin \varphi_{r, f} & 0
\end{array}\right] .
$$

Taking the estimated 3-D center coordinates and radius of the antenna edge as prior information, then the minimization is simplified as

$$
\begin{equation*}
\min _{\alpha_{a} \beta_{a}} \sum_{f=1}^{F} \frac{\left|\min \left(a_{r f}, b_{r f}\right)-\min \left(\hat{a}_{f}, \hat{b}_{f}\right)\right|}{\hat{r}_{a}}+\frac{\left|\gamma_{r f}-\hat{\gamma}_{f}\right|}{\pi / 2} \tag{24}
\end{equation*}
$$

where $0^{\circ} \leqslant \alpha_{a} \leqslant 90^{\circ}$ and $0^{\circ} \leqslant \beta_{a} \leqslant 360^{\circ}$, and the denominators $\hat{r}_{a}$ and $\pi / 2$ are used to normalize the parameters in different dimensions to balance the confidence, which aims to ensure the accuracy of estimated attitude direction.

In this paper, we adopt the classical PSO algorithm to solve the optimization. In the PSO algorithm, the particle position is the solution to minimize in Eq. 24, and it is expressed as

$$
\begin{equation*}
G=\left(\alpha_{a}, \beta_{a}\right)^{\mathrm{T}} \tag{25}
\end{equation*}
$$

The objective function of the PSO algorithm is defined as

$$
\begin{equation*}
J=\sum_{f=1}^{F} \frac{\left|\min \left(a_{r, f}, b_{r, f}\right)-\min \left(\hat{a}_{f}, \hat{b}_{f}\right)\right|}{\hat{r}_{a}}+\frac{\left|\gamma_{r, f}-\hat{\gamma}_{f}\right|}{\pi / 2} . \tag{26}
\end{equation*}
$$

A brief flow of the PSO algorithm is given as follows:
Step 1: Set the number of particles and the maximum number of iterations. Generate the initial position and velocity of each particle by randomly sampling within the solution space and a preset maximum speed.

Step 2: Calculate the objective function of each particle. Find the position of global optimal solution Gbest searched by the swarm, and the position of historical optimal solution Pbest searched by each particle.

Step 3: Update velocity and position of each particle, and the classical updating rules are $V_{i}(t+1)=\sigma_{4} V_{i}(t)+\sigma_{5}$ rand $_{1}\left(\right.$ Pbest $\left.-X_{i}(t)\right)+$

$$
\begin{align*}
& \sigma_{6} \text { rand }_{2}\left(\text { Gbest }-X_{i}(t)\right)  \tag{27}\\
& \quad X_{i}(t+1)=X_{i}(t)+V_{i}(t) \tag{28}
\end{align*}
$$

where $V_{i}(t)$ and $X_{i}(t)$ are the velocity and position of the $i$-th particle in the iteration, $\operatorname{rand}_{1}$ and $\operatorname{rand}_{2}$ are two random parameters uniformly distributed within $[0,1], \sigma_{5}$ and $\sigma_{6}$ are two learning rate weights that balance contributions of the global and local influence, $\sigma_{4}$ is the inertia weight, and a relatively small weight is better for the local search, while a large weight is better for the global search. If the algorithm reaches the maximum number of iterations or a minimum error criterion, which refers to the minimum moving distances of Gbest and Pbest, is satisfied, go to Step 4, otherwise, go to Step 2.

Step 4: Output the position of the ideal particle $\hat{G}=$ $\left(\hat{\alpha}_{a}, \hat{\beta_{a}}\right)^{\mathrm{T}}$.

Based on the PSO algorithm, the attitude direction parameters in the radar measurement coordinate system are estimated, and they can be converted to the other coordinate system according to Eq. 5.

## 3. 5 Observation LOS angle constraint

It is well known that the ISAR image shows the projection of target on the LOS plane. Thus, if the LOS elevation angle is constant during the radar observation process, the points having the same azimuth coordinates in the same plane perpendicular to the LOS angle will project to the same point in the ISAR image. In this case, it is hard to reconstruct of the 3-D geometry of target based on the sequential ISAR images. For a parabolic antenna target, if the radar LOS angle is nearly parallel to the direction of the parabolic antenna, it will lead to insufficient observation. When the observation diversity of LOS angles is significantly insufficient, the optimization function in Eq. 26 possibly leads to a wrong solution. Conventional ground-based radar is static, and the variation of radar LOS angle depends on the movement of space targets. For an HEO target, the change of the LOS elevation angle is inadequate to match up with that of the azimuth angle, which easily leads to an insufficient observation. To overcome this shortcoming, the proposed spacebased THz radar flexibly adjusts the orbital height to change the LOS elevation angle to achieve sufficient observation.

## 4 Experiment results and analysis

## 4. 1 Imaging performance comparison

In this experiment, the imaging performance between the commonly used ground-based Ku-band ISAR system and the proposed space-based THz ISAR system is compared. The main parameters of the two radar systems are listed in Table 1. The target is a parabolic antenna model, whose size parameters are the same as that in Sect. 2, and the 3-D center coordinates and attitude direction angle in the measurement coordinate system are $(-0.1 \mathrm{~m},-0.1 \mathrm{~m}, 0.2 \mathrm{~m})$ and $\left(40^{\circ}, 180^{\circ}\right)$, respectively. The observation LOS angles of two radars are the same, the elevation angle is $20^{\circ}$, and the azimuth angle ranges from $3.5^{\circ}$ to $6.5^{\circ}$. The azimuth resolution of Ku-band radar and THz radar is 17.15 cm and 1.30 cm , respectively. The equivalent observation scene is shown as Fig. 6. The noise-free VH-polarization echoes are calculated.

Figure 7 shows the ISAR imaging results of parabol-

Table 1 Main Parameters of the ISAR System表1 ISAR系统的主要参数

| Radar type | $\mathrm{Ku}-$ band radar | terahertz radar |
| :---: | :---: | :---: |
| Center frequency of signal | 16.7 GHz | 220 GHz |
| Bandwidth | 2 GHz | 10 GHz |
| Range resolution | 7.5 cm | 1.5 cm |

ic antenna model utilizing the two radars．It can be seen from Fig． 7 （a）that the Ku－band ISAR system cannot achieve fine imaging of parabolic antenna edge because of its limited range resolution and azimuth resolution． Based on this ISAR image，it is hard to detect the exis－ tence of parabolic antenna component，let alone recon－ struct the key parameters．In Fig．7（b），it is easy to identify the parabolic antenna component，and the key parameters of the projected ellipse can be directly ob－ tained based on the proposed improved RHT method in Sect．IV．

This experiment verifies the superiority of THz radar on fine imaging and target recognition of small space tar－ gets．Actually，higher frequency THz radar，such as 440 GHz and 670 GHz ，can obtain more refined ISAR imag－ ing results．Then，the parabolic antenna edge in ISAR images will also be clearer to be identified，which will further increase the estimation accuracy of attitude direc－ tion．Nonetheless，taking the current technic level of high－power THz device and complexity of electromagnet－ ic calculation into consideration，this paper only concen－ trates on the research at 220 GHz band．


Fig． 6 Equivalent observation scene of ISAR imaging图 6 ISAR成像的等效观测场景


Fig． 7 ISAR imaging results of the parabolic antenna model（a） Ku－band radar，（b）terahertz radar
图7 抛物面天线 ISAR成像结果（a）Ku频段雷达，b）太赫兹雷达

## 4． 2 Attitude direction estimation results

In this experiment，we utilize the proposed method


Fig． 8 Simplified three－dimensional satellite model图8 简化的三维卫星模型
to estimate the attitude direction of the parabolic antenna load on a space target．The target is a simplified satellite model including a parabolic antenna load，a solar panel， and a main body，as shown in Fig．8．The length and width of the solar panel is 1.6 m and 0.4 m ，respective－ ly．The size parameters of the parabolic antenna are the same as that in Section III，and the 3－D center coordinates and attitude direction angles in the measurement coordi－ nate system are $(-1 \mathrm{~m},-0.1 \mathrm{~m}, 0.2 \mathrm{~m})$ and $\left(50^{\circ}, 220^{\circ}\right)$ ， respectively．The simulation adopts a 220 GHz ISAR sys－ tem and known angle tracking data to generate a long－ time observation image sequence．The main parameters of the THz ISAR system are the same as that in Table 1. The VH－polarization echoes are calculated．The simulat－ ed sequence includes 10 RD images，and the signal－to－ noise ratio（SNR）of each image is 10 dB ．The simulated tracking LOS angle trajectory is shown as Fig．9．The echo data corresponding to the red solid line are utilized to generate the ISAR images，and the other part of the LOS angle trajectory is the orbit adjustment stage to achieve sufficient observation．


Fig． 9 Radar tracking LOS angle trajectory
图9 雷达跟踪视线角轨迹

The sequential THz ISAR imaging results with RC－ MC are shown as Fig．10．Obviously，the cross－polar－ ized ISAR images are sensitive to the edges and corners of the targets．After the image preprocessing，the solar panel component appeared as parallelogram in the ISAR images can be detected utilizing the Radon transforma－ tion method ${ }^{[31]}$ ，and the detection results are shown as Fig．11．The main purpose in detecting the solar panel component is to reduce the computation complexity in the following parabolic antenna component detection opera－ tion．It should be noted that the marked blue solid lines have been expanded by ten pixels to ensure that the solar panel component can be removed as much as possible． Remove the scattering points located on the detected area
of the solar panel，the residual scattering points are uti－ lized to detect the parabolic antenna component based on the proposed improved RHT method．The parabolic an－ tenna detection results are shown as Fig．12．In the im－ proved RHT method，the tolerances $\delta_{1}, \delta_{2}, \delta_{3}$ ，and $\delta_{4}$ are set $2 \mathrm{~mm}, 2 \mathrm{~mm}, 2 \mathrm{~mm}$ ，and 5 degrees，respectively． The score number is set 3 ，and the maximum searching number is set 50000 ．It can be seen from Fig． 12 that the parabolic antenna component in each ISAR image has been detected，and the detection results match the real shape of the projection of the parabolic antenna edge well．The detection process of parabolic antenna compo－ nent in each ISAR image is repeated by 50 times to de－ base the contingency，and the mean absolute errors of the five estimated key parameters of the ellipse in each ISAR image are shown as Fig．13．It can be seen that the maximum error of $\hat{x}_{e}, \hat{y}_{e}, \hat{a}$ and $\hat{b}$ is less than 2 cm ， and the maximum error of $\hat{\gamma}$ is less than 10 degrees．This result verifies the efficiency of the proposed improved RHT method in automatically detect the parabolic anten－ na component from sequential THz ISAR images．

Taking the parameter set $\hat{s}=\left\{\hat{x}_{e}, \hat{y}_{e}, \hat{a}, \hat{b}, \hat{\gamma}\right\}$ of each ISAR image and tracking LOS angle data as prior infor－ mation，the 3－D center coordinates and radius of the par－
abolic antenna edge are estimated based on Eq． 21 and Eq．22．Finally，according to the optimization in Eq．26， the attitude direction parameters of the parabolic antenna load are solved by the PSO algorithm．The number of par－ ticles，the maximum number of iterations，and the maxi－ mum speed of particle are set 30,100 ，and 5 ，respec－ tively．The values of $\sigma_{4}, \sigma_{5}$ and $\sigma_{6}$ are all set 1 ．The PSO algorithm is repeated by 50 times to ensure the reli－ ability of the estimated attitude direction parameters． The estimation results of the parabolic antenna parame－ ters are given in Table 2．Compared with the real parame－ ters of the parabolic antenna model，the direction estima－ tion accuracy of both elevation angle and azimuth angle is about 1 degree，and the estimation accuracy of radius is superior to 5 mm ．This experiment illustrates the effec－ tiveness and robustness of the proposed method in atti－ tude direction estimation of parabolic antenna load．

## 4． 3 Error analysis

Under ideal condition，three ISAR images are enough to accurately estimate the attitude direction． However，in the real application，there always exists some accidental errors in the attitude direction estimation process．The errors in the proposed attitude direction es－ timation process in this paper can be concluded as the fol－ lowing three aspects：


Fig． 10 Sequential terahertz ISAR imaging results
图10 序列太赫兹 ISAR成像结果


Fig． 11 Solar panel component detection results（marked with blue solid lines）
图 11 太阳能帆板部件检测结果（用蓝色实现标注）

Table 2 Estimation Results of Parabolic Antenna Parameters
表2 抛物面天线参数的估计结果

| Parameter type | True value | Estimated value | Absolute error |
| :---: | :---: | :---: | :---: |
| Center coordinates | $(-1 \mathrm{~m},-0.1 \mathrm{~m}, 0.2 \mathrm{~m})$ | $(-0.9965 \mathrm{~m},-0.1031 \mathrm{~m}, 0.1873 \mathrm{~m})$ | $(0.0035 \mathrm{~m}, 0.0031 \mathrm{~m}, 0.0127 \mathrm{~m})$ |
| Length of radius | 0.25 m | 0.2465 m | 0.0035 m |
| Attitude direction | $\left(50^{\circ}, 220^{\circ}\right)$ | $\left(50.9135^{\circ}, 221.1726^{\circ}\right)$ | $\left(0.9135^{\circ}, 1.1726^{\circ}\right)$ |



Fig． 12 Parabolic antenna component detection results（marked with blue solid lines）
图 12 抛物面天线部件检测结果（用蓝色实现标注）


Fig． 13 Absolute error of estimated ellipse parameters in each ISAR image
图13 每一幅 ISAR 图像中椭圆参数估计的绝对误差

1）Although the RD imaging process has taken the range cell migration of the target into consideration，the RD imaging results still exist position errors introduced by the approximation of the RD imaging algorithm and gridding of the ISAR images．

2）Limited by the geometric relation between radar and target，in several of the sequential THz ISAR imag－ es，the RD imaging results of the parabolic antenna load may have relatively poor image quality，which will affect the accuracy of the extracted ellipse parameters based on the improved RHT method．

3）The two－level estimation method proposed in this paper firstly estimates the center and radius of the para－ bolic antenna load．Then，taking these estimated param－ eters as priori information，the attitude direction parame－ ters are estimated based on the PSO algorithm．In this process，there exists a transferring error．

Taking these possible errors into consideration， more than three ISAR images should be obtained to en－ sure the robustness of the attitude direction estimation re－ sults．To investigate the influence of the number of ISAR images on attitude direction estimation，we estimate the attitude direction utilizing different ISAR images，and the corresponding absolute errors of the estimated atti－ tude direction parameters with 50 repetitions are shown as Fig．14．It can be seen that when the number of ISAR images is larger than 3 ，the absolute errors of both esti－ mated elevation angle and azimuth angle are below 2 de－ grees．Besides，the absolute errors of the estimated atti－ tude direction parameters are not proportional to the num－ ber of used ISAR images because of the diversity image
quality．


Fig． 14 Absolute error of attitude direction parameters图14 指向参数的绝对误差

## 5 Conclusions

To monitor and analyze the working state and poten－ tial intention of a space target，this paper takes the lead in providing a complete set of theories to estimate the atti－ tude direction of parabolic antenna loads．Both the imag－ ing system and method are novel．The proposed space－ based THz radar system can achieve successively suffi－ cient observation and high－resolution imaging of both high earth orbit satellite targets and small satellite tar－ gets．Taking the electromagnetic scattering characteris－ tics of parabolic antenna into consideration，the proposed attitude direction estimation algorithm utilizes the robust shape feature of parabolic antenna edge to replace the in－
valid point scattering center feature．Accommodating the ISAR geometric projection matrices，the attitude direc－ tion is recovered through a optimization with automatical－ ly detected shape parameters，which is solved by a two－ level estimation algorithm including the least squares es－ timation and PSO．Simulation experiments have illustrat－ ed the effectiveness and robustness of the proposed meth－ od．It should be noted that the proposed algorithm is lim－ ited to the three－axis stabilized space targets．Attitude di－ rection estimation of parabolic antenna loads on an insta－ ble satellite is our research focus in the next stage．

## References

［1］SHENG Wei－Dong，LONG Yun－Li，ZHOU Yi－Yu．Analysis of tar－ get location accuracy in space－based optical－sensor network［J］．Ac－ ta Optica Sinica（盛卫东，龙云利，周一宇．天基光学传感器网络目标定位精度分析，光学学报），2011，31（2）：255－261．
［2］PENG Hua－Feng，CHEN Jing，ZHANG Bin．Simulation study of space multi－target imaging for space－based opto－electronic tele－ scope［J］．Optical Technique（彭华峰，陈鲸，张彬．天基光电望远镜空间多目标成像模拟技术研究，光学技术），2007，33（2）： 219－222．
［3］Avent R，Shelton J．The ALCOR C－band imaging radar［J］．IEEE Antennas Propag．Mag．，1996，38（3）：16－27．
［4］Ender J，et al．Radar techniques for space situational awareness［C］． International Radar Symposium，2011：21－26．
［5］Usoff J，Leushacke L，Brenner A，et al．Haystack ultra－wideband satellite imaging radar antenna［C］．IEEE Benjamin Franklin Sympo－ sium on Microwave and Antenna Subsystems for Radar，Telecommu－ nication，and Biomedical Applications， 2016.
［6］ZHANG Ye，WU Chen－Guang，DENG Bin，et al．Terahertz SAR moving target imaging method based on the phase compensation and equivalent movement［J］．Journal of Systems Engineering and Elec－ tronics（张野，吴称光，邓彬等。基于相位补偿和等效运动的太赫兹 SAR 运动目标成像方法，系统工程与电子技术），2016， 38 （10）：2296－2302．
［7］Caris M，Stanko S，Palm S，et al． 300 GHz radar for high resolution SAR and ISAR applications［C］．16th International Radar Sympo－ sium，Dresden，Germany，2015：577－580．
［8］Cheng B，Jiang G，Cheng W，et al．Real－time imaging with a 140 GHz inverse synthetic aperture radar［J］．IEEE Trans．THz Sci．Tech－ nol．，2013，3（5）：594－605．
［9］Zhang Y，Yang Q，Deng B，et al．Estimation of translational motion parameters in terahertz interferometric inverse synthetic aperture ra－ dar（InISAR）imaging based on a strong scattering centers fusion technique［J］．Remote Sens．，2019，11（10）．
［10］Zuo F，Li J，Hu R，et al．Unified coordinate system algorithm for terahertz video－SAR image formation［J］．IEEE Trans．THz Sci． Technol．，2018，8（6）：725－735．
［11］Perrier R，Arnaud E，Sturm P，et al．Satellite image registration for attitude estimation with a constrained polynomial model［C］．In Proc． IEEE Int．Conf．Image Process．，2010：925－928．
［12］Yang X，Wen G，Zhong J，et al．A 3－D electromagnetic－model－ based algorithm for absolute attitude measurement using wideband ra－ dar［J］．IEEE Geosci．Remote Sens．Lett．，2015，12（9）：1878－1882．
［13］Lemmens S，Krag H，Rosebrock J，et al．Radar mappings for atti－ tude analysis of objects in orbit［C］，In Proc．6th Eur．Conf．Space Debris，Darmstadt，Germany，2013：20－24．
［14］CAO Xing－Hui，SONG Qing－Lei，JIANG Yan，et al．Interferomet－ ric ISAR 3D imaging of target satellite in low earth orbit［J］．Radar Science and Technology（曹星慧，宋庆雷，姜岩，等。低轨卫星目标干涉 ISAR 三维成像方法，雷达科学与技术），2007，5（3）： 204－208．
［15］Mcfadden E，et al．Three－dimensional reconstruction from ISAR se－ quences［C］．In Proc．Aerosense Int Soc．Opt．Photon．，2002： 58 $-67$.
［16］Zhou Z，Du R，Liu L，et al．A novel method of three－dimensional geometry reconstruction of space targets based on the ISAR image se－ quence［C］．In 6th Asia－Pacific Conference on Synthetic Aperture Radar， 2019.
［17］Ferrara M，Arnold G，Stuff M，Shape and motion reconstruction
from 3D－to－1D orthographically projected data via object－image re－ lations［J］．IEEE Trans．Pattern Anal．Mach．Intell．，2009，31（10）： 1906－1912．
［18］Liu L，Zhou F，Bai X，et al．Joint crossrange scaling and 3D geome－ try reconstruction of ISAR targets based on factorization method［J］． IEEE Trans．Image Process．，2016，25（4）：1740－1750．
［19］Wang F，Xu F，Jin Y．Three－Dimensional Reconstruction From a Multiview Sequence of Sparse ISAR Imaging of a Space Target［J］． IEEE Trans．Geosci．Remote Sens．，2018，56（2）：611－620．
［20］Yang S，Jiang W，Tian B．ISAR Image Matching and 3D Reconstruc－ tion Based on Improved SIFT Method［C］．In International Confer－ ence on Electronic Engineering and Informatics，2019：224－228．
［21］Hartley R，Zisserman A，Jin T．Multiple view geometry in computer vision［M］，Cambridge，MA，USA ：Cambridge，2003：244－250．
［22］Tomasi C，Kanade T．Shape and motion from image streams under or－ thography：A factorization method［J］．Int．J．Comput．Vis．，1992， 9 （2）：137－154．
［23］WANG Xin，GUO Bao－Feng，SHANG Chao－Xuan．3D reconstruc－ tion of target geometry based on 2D data of inverse synthetic aperture radar images［J］．Journal of Electronics and Information Technology （王昕，郭宝峰，尚朝轩。基于二维 ISAR 图像序列的雷达目标三维重建方法，电子与信息学报），2013，35（10）：2475－2480．
［24］Y in H，Huang P．Further comparison between two concepts of radar target angular glint［J］．IEEE Trans．Aerosp．Electron．Syst．，2008， 44（1）：372－380．
［25］Liu T，et al．Wide－angle CSAR imaging based on the adaptive subap－ erture partition method in the terahertz band［J］．IEEE Trans．THz Sci．Technol．，2018，8（2）：165－173．
［26］Zhou Y，Lei Z，Cao Y ，et al．Attitude estimation and geometry recon－ struction of satellite targets based on ISAR image sequence interpre－ tation［J］．IEEE Trans．Aerosp．Electron．Syst．，2018，55（4）：1698－ 1711.
［27］Kennedy J，Eberhart R．Particle swarm optimization［C］．Proc．IEEE Int．Conf．Neural Netw．，2002：1942－1948．
［28］Montenbruck O，Gill E．Satellite orbits：Models，methods and appli－ cations［M］．Cambridge，MA，USA：Springer， 2012.
［29］Li X，Liu G，Ni J，et al．Autofocusing of ISAR images based on en－ tropy minimization［J］．IEEE Trans．Aerosp．Electron．Syst．，1999， 35（4）：1240－1252．
［30］Wang J，Kasilingam D．Global range alignment for ISAR［J］．IEEE Trans．Aerosp．Electron．Syst．，2003，39（1）：351－357．
［31］Martorella M．Novel approach for ISAR image cross－range scaling ［J］．IEEE Trans．Aerosp．Electron．Syst．，2008，44（1）：281－294．
［32］Xing M，Wu R，Lan J，et al．Migration through resolution cell com－ pensation in ISAR imaging［J］．IEEE Geosci．Remote Sens．Lett．， 2004，1（2）：141－144．
［33］Liang M，et al．Micro－Doppler characteristics of sliding－type scatter－ ing center on rotationally symmetric target $[\mathrm{M}]$ ．Science China Infor－ mation Science，2011，54（9）：1957－1967．
［34］Keller J．Geometrical theory of diffraction［J］．J．Optics Soc of Ameri－ ca．，1962，52（2）：116－130．
［35］Kline M，Kay I．Electromagnetic theory and geometrical optics［M］． J．Wiley and Sons，New York，1965：124－144．
［36］James G．Geometrical theory of diffraction for electromagnetic waves ［M］．Peter Peregrinus Ltd．England，1981：96－113．
［37］Wang F，Thomas F，Jin Y．Simulation of ISAR imaging for a space target and reconstruction under sparse sampling via compressed sens－ ing［J］．IEEE Trans．Geosci．Remote Sens．，2015， 53 （6）：3432－ 3441.
［38］Yusuf U．Modified theory of physical optics approach to wedge dif－ fraction problems［J］．Optics Expre．，2005，13（1）：216－224．
［39］Liu Z，Cui T，Xing Z，et al．Electromagnetic scattering characteris－ tics of PEC targets in the terahertz regime［J］．IEEE Antennas Propag．Mag．，2009，51（1）：39－50．
［40］Deans S．The radon transform and some of its applications［M］．North Chelmsford，MA，USA ：Courier Corporation，2007：10－30．
［41］Priyanka M．A survey of Hough transform［J］．Pattern Recog．，2015， 48（3）：993－1010．
［42］Xu L，Oja E，Kultaned P．A new curve detection method：Random－ ized Hough Transform（RHT）［J］．Pattern Recog．Lett．，1990， 11 （5）：331－338．
［43］CHEN Yan－Xin，QI Fei－Hu．A new ellipse detection method using randomized Hough transform［J］．J．Infrared Millim．Waves（陈燕新，戚飞虎。一种新的基于随机 Hough 变换的椭圆检测方法。红外与毫米波学报），2000，19（1）：43－47．

