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THEORETICAL ANALYSIS OF A NEW GRATING LEAKY WAVE ANTENNA BASED ON LEFT-HANDED MATERIALS

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Abstract: A new grating leaky wave antenna based on left-handed material was proposed and its radiation characteristics were carefully and rigorously analyzed by using a method which combined the rigorous mode matching procedure with multimode network method. The variations of the leakage constant with structure parameters were given in this study to show the special properties of this kind of antenna. The numerical results indicate that the new leaky wave antenna has much larger leakage constant than that of conventional one. The reason for this phenomenon is discussed.

Key words: left-handed material (LHM); periodic structure; leaky wave grating antenna
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一种基于左手介质的新型介质栅漏波天线的理论分析

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摘要:提出了一种基于左手介质的新型介质栅漏波天线,并采用多模网络与严格模匹配相结合的方法,对该左手介质栅天线的辐射特性进行了仔细严格的分析. 文中给出了漏波系数随天线结构参数变化的关系以显示该左手介质天线特有的性质. 数值结果表明这种新型漏波天线比传统介质栅天线具有更强的辐射能力. 讨论了产生这种现象的原因.

关 键 词:左手介质;周期结构;栅漏波天线

Introduction

Left-handed material (LHM) is a material whose permittivity and permeability are simultaneously negative, which has been theoretically analyzed by Veselago in 1968 [1]. In this kind of medium, the phase velocity and the group velocity of the propagation wave are in the anti-parallel direction[2]. Thus, the conventional material whose permittivity and permeability are simultaneously positive is called right-handed material (RHM). Recently, the LHM and its applications attract more and more attention and some significant idea has been brought forward and carefully studied [3,4].

As we know, the leaky wave antenna consisting of dielectric periodic structure has many advantages, such as wide operating band, simple in structure, easy for design and fabrication, high directivity and scanable electronically by changing frequency[5,6]. However, the earlier investigation has indicated that the leakage of the conventional grating leaky wave antenna is small so

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that a long antenna structure is required for getting reasonably high efficiency to satisfy the practical applications. Some researches have been done to increase the radiation strength of the grating leaky wave antenna[6~8]. The appearance of LHM gives a new alternative to improve the performance of the grating antenna.

In this paper, a new grating leaky wave structure based on LHM is presented and the dependence of the radiation characteristics on related parameters of the leaky wave antenna is analyzed for the first time. Also, some interesting phenomenon has been found and the reason for the phenomenon is explained.

1 Analysis

Fig. 1 shows the structure and related parameters under consideration. It consists of several parts including semi-infinite air region, dielectric-air periodic slab layer and uniform substrate. The substrate and the dielectric slabs in the periodic layer compose of the same LHM. This kind antenna can be excited by the mode in grounded LHM slab and with a tapering corrugation section to reduce the VSWR.

It is well known that the radiation characteristics of the leaky wave antenna can be completely determined by its complex transmission constant k_x. To find out the k_x, three steps are taken; first, the eigenfunctions are determined for every layer, including air, periodic medium and substrate regions; second, the boundary condition of electromagnetic fields is used at each interface to obtain the transfer relation of the admittance matrix between the layers. Finally, the generalized transverse resonant method is used to determined k_x.

To determine the eigenfunctions of the periodic slab, the structure shown in Fig. 2 (a) is analyzed,

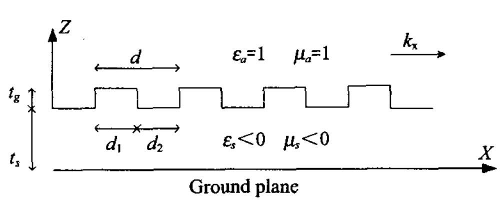

Fig. 1 The structure of LHM periodic grating leaky wave antenna and related parameters.
图1 左手介质栅漏波天线的结构和相关参数

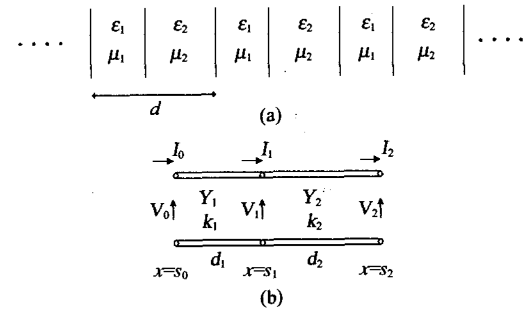

Fig. 2 (a) Infinite periodic dielectric layers. (b) Equivalent transmission line model of unit cell.
图2 (a)无界周期介质阵列 (b)周期单元等效传输线模型

which is an unbounded periodic array of dielectric slabs. Fig. 2 (b) shows the equivalent circuit for a unit cell.

From the transmission line theory, the dispersion equation may be written as:

$$\cos(k_x d) = \cos(k_1 d)\cos(k_2 d) - 1/2[(Z_1/Z_2) + (Z_2/Z_1)]\sin(k_1 d)\sin(k_2 d) \quad , \quad (1)$$

for TM, $Z_i = 1/Y_i = k_i/\omega\varepsilon_0\varepsilon_i, k_i^2 = k_0^2\varepsilon_i\mu_i - k_z^2, i = 1,2$. Suppose that one of the roots is $\lambda_1 = e^{-jk\times d}$, then the another root should be $\lambda_2 = e^{jk\times d}$, where k_x is the transmission constant of Floquet mode.

In the transmission line network, for each eigen excitation, the voltage and current distribution ($V(x)$ and $I(x)$) of the transmission line can be determined. It is clear that $V(x)$ and $I(x)$ are periodic functions with period d, therefore they can be expanded in Fourier series,

$$V(x) = \sum_{n=-\infty}^{+\infty} V_n e^{-jk_{xn}x}, \quad I(X) = \sum_{n=-\infty}^{+\infty} I_n e^{-jk_{xn}x} \quad , (2)$$

where $k_{xn} = k_{x0} + 2n\pi/d$. k_{x0} and k_{xn} are the transmission constant of the foundational and the n-th Floquet mode respectively. V_n and I_n are the amplitudes of the n-th space harmonic of $V(x)$ and $I(x)$, which can be obtained as did in [7] and is omitted here for simplicity.

From above analysis, it can be seen that the eigen solutions of the periodic structure satisfy the Floquet's condition, and can be represented as the summation of space harmonics, which are generally exited everywhere in whole structure; the eigenfunctions in the air and substrate regions should be represented by infinite number of transmission lines; each line stands for one

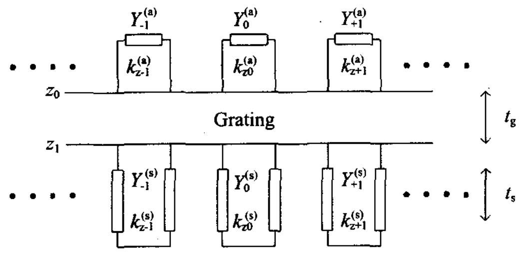

Fig. 3 Equivalent multimode network representation of the grating antenna.
图 3 介质栅结构的等效网络模型

space harmonic. Thus, the whole antenna can be described by the multimode network shown in Fig. 3. It is easy to determine the admittance matrices looking into the homogeneous medium.

At the interface $z = z_0$, the admittance matrix Y_a looking into the air region, is a diagonal matrix

$$\mathbf{Y}_a = (\delta_{nl} Y_n^{(a)}) \quad , \tag{3}$$

$Y_n^{(a)} = \omega\varepsilon_0 / k_{zn}^{(a)}$, $k_{zn}^{(a)2} = k_0^2 - k_{xn}^2$, $k_{xn} = k_{x0} + 2n\pi/d$. Similarly, at the $z = z_1$ interface, the admittance matrix $\mathbf{Y}_s$, looking down into the uniform substrate region, is also diagonal matrix,

$$\mathbf{Y}_s = (-\delta_{nl} j Y_n^{(s)} \cot(k_{zn}^{(s)} t_s)) \quad , \tag{4}$$

$Y_n^{(s)} = \omega\varepsilon_0\varepsilon_s / k_{zn}^{(s)}$, $k_{zn}^{(s)2} = k_0^2\varepsilon_s\mu_s - k_{xn}^2$, $k_{xn} = k_{x0} + 2n\pi/d$. Since the eigenmodes of the periodic structure have been determined, the transfer relationship of admittance matrix for each layer can be obtained by using the boundary condition of the electromagnetic fields.

After $Y_{nm}^{(a)}$ which is an element of the output matrix $\mathbf{Y}_a$ of the air region is known, the electromagnetic field in the air layer can be expressed in vector form:

$$\mathbf{H}_{tn}^{(a)}(z_0) = \sum_{m=-\infty}^{+\infty} Y_{nm}^{(a)} [\mathbf{i}_z \times \mathbf{E}_{tm}^{(a)}(z_0)] \quad . \tag{5}$$

Since the transverse component should be continuous at $z = z_0$ interface, therefore electric and magnetic fields in the periodic layer should satisfy the following equation.

$$\mathbf{H}_{tn}(z_0) = \sum_{m=-\infty}^{+\infty} Y_{nm}^{(a)} [\mathbf{i}_z \times \mathbf{E}_{tm}(z_0)] \quad , \tag{6}$$

where $\mathbf{E}_{tn}$ and $\mathbf{H}_{tn}$ represent respectively the n-th space harmonic electrical and magnetic fields in the periodic layer. Similarly, we have the following equation at $z = z_1$,

$$\mathbf{H}_{tn}(z_1) = \sum_{n=-\infty}^{+\infty} Y_{nm}^{(1)} [\mathbf{i}_z \times \mathbf{E}_{tm}(z_1)] \quad . \tag{7}$$

$Y_{nm}^{(1)}$ is an element of the output matrix $\mathbf{Y}_1$ at $z = z_1$ interface, looking up to the periodic layer. The transverse fields in the periodic region $z_0 \leqslant z \leqslant z_1$ for either TE or TM polarization can be expressed as modal summation with suppressing the constant vectors

$$E_t(x,z) = \sum_{l=-\infty}^{+\infty} V_l(x) [a_l e^{-jk_{zl}z} + b_l e^{jk_{zl}z}] \quad , \tag{8}$$

$$H_t(x,z) = \sum_{l=-\infty}^{+\infty} I_l(x) [a_l e^{-jk_{zl}z} - b_l e^{jk_{zl}z}] \quad . \tag{9}$$

The n-th Fourier component is

$$E_{tn}(z) = \sum_{l=-\infty}^{+\infty} V_{nl} [a_l e^{-jk_{zl}z} + b_l e^{jk_{zl}z}] \quad , \tag{10}$$

$$H_{tn}(z) = \sum_{l=-\infty}^{+\infty} I_{nl} [a_l e^{-jk_{zl}z} - b_l e^{jk_{zl}z}] \quad , \tag{11}$$

where a_l and b_l are respectively the forward and backward traveling wave amplitude of l-th Floquet mode in the periodic layer. V_{nl} and I_{nl} are the amplitudes of the n-th space harmonic of $V_l(x)$ and $I_l(x)$. The output reflection coefficient matrix $\mathbf{\Gamma}$ at lower surface of the $z = z_0$ interface can be expressed:

$$e^{j\mathbf{k}z_0}\mathbf{b} = \mathbf{\Gamma} e^{-j\mathbf{k}z_0}\mathbf{a} \quad , \tag{12}$$

$$e^{j\mathbf{k}z_0} = (\delta_{nl} e^{j\mathbf{k}_{zl}z_0}) \quad , \tag{13}$$

$$e^{-j\mathbf{k}z_0} = (\delta_{nl} e^{-jk_{zl}z_0}) \quad , \tag{14}$$

$$\mathbf{a} = [\cdots \quad a_1 \quad a_0 \quad a_{-1} \quad \cdots]^{\mathrm{T}}$$
$$\mathbf{b} = [\cdots \quad b_1 \quad b_0 \quad b_{-1} \quad \cdots]^{\mathrm{T}} \quad . \tag{15}$$

Substituting (13) ~ (15) into (12), then (12) into (6), invoking (10) (11), we get

$$\mathbf{\Gamma} = (\mathbf{I} + \mathbf{Y}_a\mathbf{V})^{-1}(\mathbf{I} - \mathbf{Y}_a\mathbf{V}) \quad . \tag{16}$$

Similarly, Substituting (10) ~ (16) into (7), we can obtain the input admittance matrix $\mathbf{Y}_1$ at $z = z_1$ interface, looking up into the upper region

$$\mathbf{Y}_1 = \mathbf{I}[1 - e^{-j\mathbf{k}t_g}\Gamma e^{-j\mathbf{k}t_g}][1 - e^{-j\mathbf{k}t_g}\Gamma e^{-j\mathbf{k}t_g}]^{-1}(\mathbf{V})^{-1} \quad , \tag{17}$$

where 1 stands for the infinite unit matrix. The admittance matrix $\mathbf{Y}_s$ at $z = z_1$ looking down into the substrate region is given by (4). Therefore, the complex eigen value of the antenna boundary value problem can be determined by the generalized transverse resonance condition at $z = z_1$ interface,

$$\det[\mathbf{Y}_1 + \mathbf{Y}_s] = 0 \quad . \tag{18}$$

The last equation defines the dispersion relation of the whole dielectric antenna, from which the complex transmission constant $k_x = \beta - j\alpha$ can be obtained. Once the complex transmission constant $k_x = \beta - j\alpha$ is obtained, the radiation characteristics of the antenna

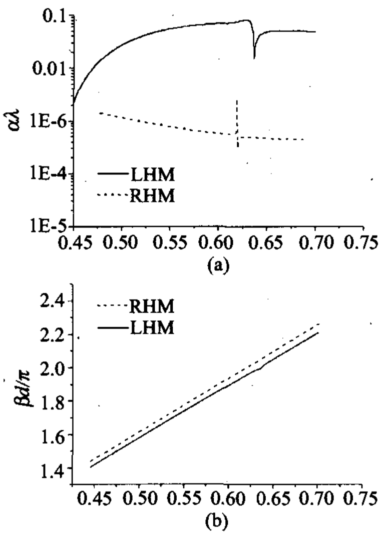

Fig. 4 (a) Variation of leakage constant with grating period. (b) variation of phase constant with grating period. $d_1 = d_2 = 0.5d$, $t_s = 0.45\lambda$, $t_g = 0.05\lambda$ ($\lambda = 1.0$).
图4 (a)漏波系数与介质栅周期长度的关系 (b) 相位常数与介质栅周期长度的关系

can be completely determined by β and α [8].

2 Numerical results

Though the LHM can be realized only when μ and ε are frequency dependent, yet, we would like to know the effect of the structure parameters on the radiation characteristics of the antenna at a fixed frequency, therefore we may assume $\mu = -1$ $\varepsilon = -2.8$ and $\mu = 1$ $\varepsilon = 2.8$ being constant for LHM and RHM respectively.

Let us discuss the effect of the period on the leaky property first. Fig. 4 (a) shows the dependence of the leakage constant on grating period for both LHM and the conventional grating antennas, assuming $t_s = 0.45\lambda$, $t_g = 0.05\lambda$, $d_1 = d_2 = 0.5d(\lambda = 1.0)$, which surely makes the existence of the fundamental mode in the LHM and RHM grating structure, though the fundamental mode for LHM and LRH structure is different[3]. The Bragg reflection in the radiation region appears at the $d/\lambda = 0.63$ for the LHM antenna, and $d/\lambda = 0.62$ for RHM antenna, where the stopband shows up and the radiation disappears. The leakage constant $\alpha\lambda$ of the LHM antenna is in the order 0.1 over a large range of d/λ values, while the $\alpha\lambda$ of conventional one is in the order of 0.001. It means that the leakage constant of the LHM antenna is more than one order larger than that of the RHM antenna. Fig. 4 (b) shows the dependence of the phase constant on grating period. It can be seen that not large phase difference is there between the LHM and RHM antenna.

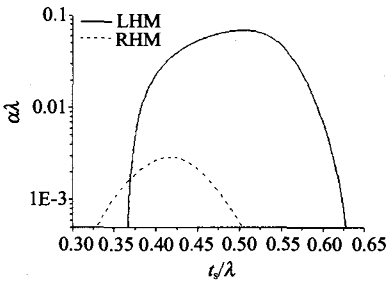

Fig. 5 Variation of leakage constant with substrate thickness. $tg = 0.05\lambda$, $d_1 = d_2 = 0.5d$, $d = 0.6\lambda$ ($\lambda = 1.0$)
图5 漏波系数与基板厚度的关系

Fig. 5 shows the dependence of the leakage constant on substrate thickness t_s, with $d = 0.6\lambda$, $t_g = 0.05\lambda$, $d_1 = d_2 = 0.5d$ ($\lambda = 1.0$). The variation of curve is as expected, because t_s strongly influences on the field distribution; at low frequency most of the energy is distributed in the air region, while at high frequency the field is mainly confined within the substrate region. In these two extreme cases the field in the periodic layer region is expected to be very weak and so is the radiation. The maximum of the leakage constant which appears at the $t_s/\lambda = 0.51$ for the LHM antenna is much larger than that for the conventional one, which appears at $t_s/\lambda = 0.42$.

The radiation constant $\alpha\lambda$ as a function of the grating thickness is shown in Fig. 6. It can be found for both antennas that when $t_g = 0$, the antenna becomes a uniform planar waveguide; no radiation occurs. As we know that the fields of a surface wave supported by a uniform planar structure are evanescent in the air region. When t_g is small, the perturbation is in the strong field region, a small increase of t_g causes great effect on radiation, but when t_g is increasing further, the additional material appears in a region where the fields are exponentially small and would have very little effect on the radiation and the curves reach a saturation circumstance. Again, it can be seen that the leakage constant of the LHM antenna is much larger than that of the RHM one.

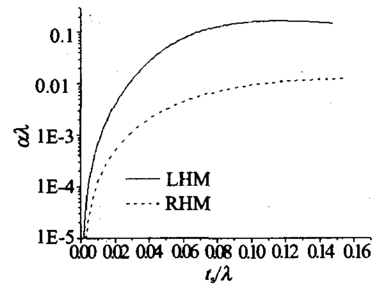

Fig. 6 Variation of the leakage constant with grating thickness. $tg = 0.4\lambda$ $d_1 = d_2 = 0.5d$, $d = 0.6\lambda$ ($\lambda = 1.0$)
图 6 漏波系数和栅高度的关系

Table 1 The amplitudes of the $n = -1$ space harmonic for different eigen modes
表 1 不同本征模的 $n = -1$ 次空间谐波幅度

eigen mode order	-3	-2	-1	0	1
Amplitude of $n = -1$ space harmonic (RHM)	0.249	0.006	0.076	3.95	0.009
Amplitude of $n = -1$ space harmonic (LHM)	0.400	0.098	5.47	5.50	0.084

Table. 1 shows the amplitudes of $n = -1$ space harmonic for different eigen modes in LHM antenna and the RHM one. It can be found that the amplitudes of the $n = -1$ space harmonic in the eigensolutions of LHM grating layer is much larger than that of the RHM one. That is just the reason why the leakage constant of the LHM grating antenna is much larger than that of the conventional one.

3 Conclusion

A new grating leaky wave antenna based on left-handed material is presented for the first time and the radiation characteristics of the antenna are carefully and rigorously analyzed by using a method which combines the mode matching procedure with multimode network method. The variations of the leakage constant with structure parameters are given in this paper to show the special properties of this new kind of antenna. The numerical results indicate that the present LHM leaky wave antenna is of much larger leakage constant than that for the conventional one. The enhancement of radiation of the LHM antenna is due to that the amplitudes of the $n = -1$ space harmonic in the eigensolutions of LHM grating layer is much larger than that of the RHM one. These properties of the LHM antenna are desirable in some applications.

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