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EFFICIENT SCHEME FOR DETERMINING FRACTAL SCALELESS RANGE

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Abstract: The terrain model based on fractal character can sufficiently represent the statistical texture features of the terrain. Better model and higher resolution DEM (Digital Elevation Model) data can be obtained from the lower resolution data by using fractal interpolation. The determination of fractal scaleless range is very important for computing the fractal characteristic parameters and modeling the digital elevation. The traditional method for determining the fractal scaleless range usually adopts the mutual test between people and compute. However, this method tends to be too subjective. Hence, we offer an auto-determining method, which has been proved to be concise, effective and efficient by experimental results.

Key words: digital elevation model (DEM); fractional Brownian motion (FBM); scaleless range
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确定分形无标度区的有效算法

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摘要:分形地貌的分维模型能够充分反映地形的统计纹理特征,利用分形内插能从低分辨率的卫星或航空照片高程数据获取高分辨率的高程数据,达到较理想的建模效果.而分形模型中无标度区的确定直接影响到分形特征参数的求解和DEM建模质量的好坏,常用方法是采用人机交互试验来选取无标度区,缺乏客观标准.本文提出一种自动确定无标度区的方法,实验结果表明,该方法在DEM自动建模过程中是简洁、有效的.

关 键 词:数字高程模型(DEM);分形布朗运动(FBM);无标度区

Introduction

Modeling the digital elevation based on computer vision is impossible to obtain the elevation data at every spot because of the problems of sheltering and error matching during the image processing. Therefore, interpolation computing of surface data is needed in order to gain high-resolution elevation data.

The theoretic analysis and experimental results[1,2,3] have already indicated clearly that the surf interpolation technology based on fractal geometry can reflect the statistic texture character about terrain and obtain a better model, while the traditional interpolation arithmetic based on smoothness-restricted can not reflect the coarse terrain.

Fractal character parameter, which only exists in a certain section, must be obtained during the process of DEM fractal interpolation. In this paper, a new method for determining fractal scaleless range has been researched. The experimental results show that this

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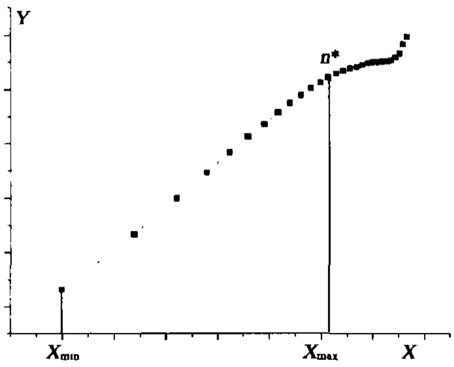

Fig. 1 Fractal features fit
图1 分形特征拟合

method is concise, effective and efficient.

1 Modelling digital elevation based on FBM

FBM (Fractional Brownian Motion) proposed by Mandelbrot and Ness[4,5] is a Gaussian random function with zero mean. The FBM function $f(x)$ is real-valued random function. According to FBM statistical similarity, for all x and Δx,

$$E[\,|f(x+\Delta x)-f(x)|\,]\cdot\|\Delta x\|^{-H} = C = 2\sigma/\sqrt{2\pi}, \qquad (1)$$

where x represents a point in the E-dimensional Euclidean space, H is a Hurst constant ($0<H<1$) for 3D terrain surfaces. Equation (1) is equivalent to

$$E[\,|f(i+\Delta i,j+\Delta j)-f(i,j)|\,]\cdot\|\Delta x\|^{-H} = C = 2\sigma/\sqrt{2\pi}. \qquad (2)$$

Taking logarithm at both sides of the equation, we get

$$\log E[\,|f(i+\Delta i,j+\Delta j)-f(i,j)|\,] - H\log\|\Delta x\| = \log C. \qquad (3)$$

According to the least square method, H and C (or σ) can be computed by fitting method in fractal scaleless range.

Provided i and j are odd numbers, the digital elevation $f(i,j)$ has already been determined. First, we can compute the digital elevation $f(i,j)$ such that both i and j are even numbers by using the folloiwng equation:

$$f(i,j) = \frac{1}{4}\{f(i-1,j-1)+f(i+1,j-1)+f(i-1,j+1)+f(i+1,j+1)\} + \sqrt{1-2^{2H-2}}\,\|\Delta x\|^{H}\sigma\cdot Gauss(). \qquad (4)$$

Then we can compute the digital elevation $f(i,j)$ such that only one of i and j is an odd numbers by equation:

$$f(i,j) = \frac{1}{4}\{f(i,j-1)+f(i+1,j)+f(i-1,j)+f(i,j+1)\} + 2^{-H/2}\sqrt{1-2^{2H-2}}\,\|\Delta x\|^{H}\sigma\cdot Gauss(), \qquad (5)$$

where, Gauss ( ) is a Gaussian random variable with $N(0,1)$ distribution. Iterating equations (4) and (5), we can get the DEM with certain quality.

The position of $f(i+\Delta i,j+\Delta j)$ can be chosen by the method presented in reference [3] and [4]. In order to simplify the calculation of practical applications, we can use Street Distance ($\|\Delta x\| = |\Delta i| + |\Delta j|$) or Chessboard Distance ($\|\Delta x\| = \max|\Delta i|,|\Delta j|$) to determine the value of $\|\Delta x\|$.

2 The method of determining scaleless range

A lower limit $\|\Delta x\|_{min}$ and an upper limit $\|\Delta x\|_{max}$ always exist in natural fractal, and can be described by scaleless range or unchanged scale range. Fractalness only exists in scaleless range. So we can make fractal analysis only in scaleless range, otherwise we cannot get the exact fractal characteristic parameters.

The traditional method to determine the scaleless range is to draw the data points to be fitted in a plane, and then find the certain linear sections by human eye. Then according to the scaleless range, fractal parameters can be computed. This method, which obviously lacks objectivity, is inconvenient for the auto-model-

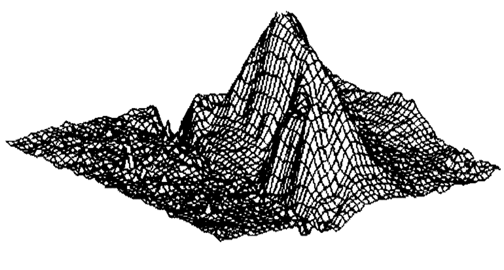

Fig. 2 The original DEM
图2 原始 DEM

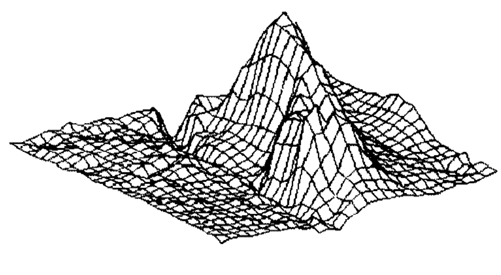

Fig. 3 The sampled DEM
图3 抽样后的 DEM

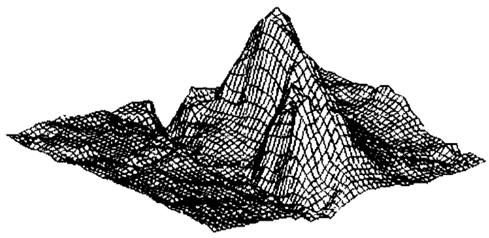

Fig. 4 The linear interpolation DEM
图 4 线性插值后的 DEM

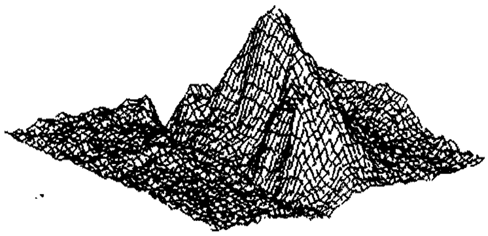

Fig. 5 The fractal interpolation DEM
图 5 分形插值后的 DEM

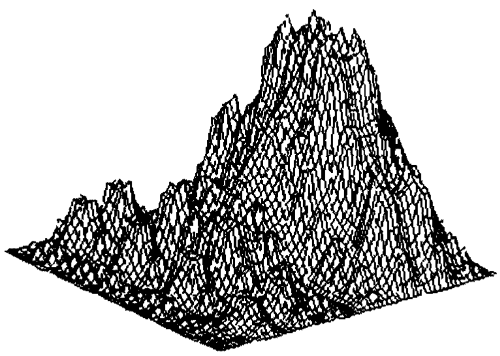

Fig. 6 The fractal DEM
图 6 分形 DEM

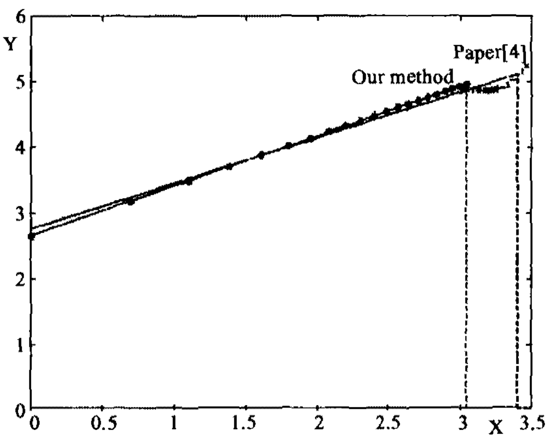

Fig. 7 The fractal fitting
图 7 分形拟合

ing analysis of DEM, because repeated man - machine interactiveness is needed to determine the scalelee range. So far, few researches have concerned the scaleless range. Currently the basic arithmetic of the most popular method in application is presented in reference [4], $\|\Delta x\|_{max} = \|\Delta x\|_{min} + n^* - 1$, where $n^* = \min\{n \mid I_{n-1} < I_n \& I_n > I_{n+1}; n \geqslant 4\}$

$$I = \frac{\sqrt{4\mu_{11}^{2} + (\mu_{20} - \mu_{02})^{2}}}{\mu_{20} + \mu_{02}},$$

and $\mu_{ij}(0 \leqslant i, j \leqslant 2; i + j = 2)$ represents variances and covariance, or the second-order central moments of a set of points in a plane. The arithmetic is complicated and the amount of computation is huge.

Reference [6] has put forward a three-line method to compute the fractal dimension for CDP record, but cannot analyze the complex terrain correctly. In the following, we will propose a linear - polynomial method, which can perform a precise analysis to the real terrain.

According to Equation (3), the data points can be fitted as $(x_i, y_i), (i = 1, 2, \cdots, n)$. Figure 1 shows the point sets (X, Y) in a plane of fractal plots, where $X = \log\|\Delta x\|$, $Y = \log(E|\Delta f(x)|)$. These points are always divided into two parts: linear and non-linear parts. Based on reference [1] and [4], we let $\|\Delta x\|_{min} = 1$ (unit length), and assume that $[1, n^*]$ $(1 < n^* \leqslant n)$ is the scaleless range which we want to determine. According to the fractal features, we can fit these points by using linear and non - linear arithmetic respectively.

a) Linear fitting $(1 \leqslant i \leqslant n^*)$: by assuming a line $v = au + b$, and fitting the line, the error square sum is:

$$\delta_1(n^*) = \sum_{i=1}^{i=n^*}(ax_i + b - y_i)^2 \tag{6}$$

b) Non-linear fitting $(n^* \leqslant i \leqslant n)$: Theoretically, we can use a high-order polynomial to fit the non-linear part. Assume a polynomial $v = \varphi(u, m)$, where m is the polynomial order. By fitting it, the error square sum is:

$$\delta_2(n^*, m) = \sum_{i=n^*}^{n}(\varphi(x_i, m) - y_i)^2 \tag{7}$$

The bigger m is, the smaller δ_2 is. If m increases, the non-linear parts will increase. Correspondingly, the linear parts will decrease, in which the fractal law is submitted very strictly. In practice, this paper limits m to a range of [1,3].

According to equations (6) and (7), all error square sums are:

$$\delta(n^*,m) = \delta_1(n^*) + \delta_2(n^*,m) \quad (8)$$

Giving m,n^* can be computed by

$$n^* = \max\{n^* \mid \arg\min(\delta(n^*,m))\}. \quad (9)$$

n^* maybe have several values different from m. The bigger n^* is, the bigger the linear parts are, and the smaller the non-linear parts are. Choosing the maximum value from all n^*, the fractal plots are the widest fractal scaleless range. Using the computed results, we can evaluate the statistical fractalness of the DEM within a range of scaleless $[1,n^*]$.

3 Results

Figure 2 is a real DEM in Three-Gores, China, and $\|\Delta i\|_{\min} = \|\Delta j\|_{\min} = 20m$, $f(i,j)_{\min} = 97.0m$, $f(i,j)_{\max} = 245.1m$. Figure 3 is the 1/2 sampled DEM of Figure 2. Figure 4 is the result of Figure 3 by using linear interpolation and Figure 5 is the result of Figure 3 by using fractal interpolation.

Table 1 shows the values of H and σ from Figure 2 to Figure 5 by using our method. Table 2 shows the values of H and σ from Figure 2 to Figure 5 by using paper [4].

From table 1 and table 2, the errors of H and σ between the original DEM and the interpolation DEM by our method are less than those of ref. [4]'s method. And the results show that the DEM after reconstruction by linear interpolation is much smoother than that by fractal interpolation, which cannot reflect the natural terrain detail character.

Table 1 The values of H and σ from figure 2 to figure 5 by using our method

表1 用本文方法计算DEM(从图2到图5)的H和σ值

	Origin DEM ($n^*=13$)	Sample DEM ($n^*=7$)	Fractal Interpolation DEM ($n^*=13$)	Linear Interpolation DEM ($n^*=13$)
H	0.7976	0.7841	0.8034(+0.0058)	0.8285(+0.0309)
σ	5.0945	8.9133	4.8843(−0.2102)	4.7819(−0.3126)

Table 2 The values of H and σ from figure 2 to figure 5 by using paper[2] method

表2 用文献[2]计算DEM(从图2到图5)的H和σ值

	Origin DEM ($n^*=14$)	Sample DEM ($n^*=7$)	Fractal Interpolation DEM ($n^*=14$)	Linear Interpolation DEM ($n^*=14$)
H	0.8031	0.7841	0.8136(+0.0105)	0.8414(+0.0383)
σ	5.1201	8.9133	4.8280(−0.2921)	4.7124(−0.4077)

Figure 6 is a DEM with fractal feature. Figure 7 gives the comparisons of analyzing the fractal scaleless range by using our and ref. [4]'s method. The results show that ref. [4]'s method extends the scaleless range, while our method limits the fractal scaleless range in the line part. In our method, the scaleless range is [1,21], the fitting coefficient and errors are 0.9991 and 0.0138. In ref. [4]'s method, the scalelee range is [1,30], the fitting coefficient and errors are 0.9907 and 0.1876.

4 Conclusion

We have presented a new scheme to determine fractal scaleless range. By using the Linear-Polynomial method in this paper, we can more statistically analyze the real complex terrain. Our experimental results show that this new method can reflect the terrain texture feature, and the arithmetic is simple and efficient.

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