

ANALYSIS ON DISPERSION CHARACTERISTICS OF PHOTONIC CRYSTAL FIBER *

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Abstract The dispersion properties of photonic crystal fiber (PCF) was analyzed by an effective refractive model, and the eigenvalue equation of the fundamental space-filling mode in infinite air silica micro-structure was presented. The analysis results show that PCF has unusual dispersion property, which can support single mode transmission in a very large wavelength range and have anomalous dispersion under single mode operation. Zero dispersion wavelength can be shifted by adjusting the structure parameters of PCF. Finally, PCF with large air holes and its applications in dispersion compensation were also discussed.

Key words photonic crystal fiber (PCF), chromatic dispersion, Holey fiber, Micro-structured fiber.

光子晶体光纤的色散特性分析 *

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摘要 采用有效折射率模型分析了光子晶体光纤的色散特性, 并给出了无限大空气玻璃微结构中基模的本征方程. 分析结果表明光子晶体光纤具有奇异的色散特性, 能在极大的波长范围内支持单模传输, 在单模工作时可以具有反常波导色散. 同时通过调整光子晶体光纤的结构参数(包括空气孔径和孔间距)可以移动零色散点的位置. 最后讨论了大空气孔光子晶体光纤的特性及其在色散补偿中的应用.

关键词 光子晶体光纤, 色散, 有孔光纤, 微结构光纤.

Introduction

Recently, photonic crystal fiber (PCF), also called holey fiber or micro-structured fiber or silica fiber with a hexagonal array of air holes along its length, has attracted a lot of attentions, since it provides many unusual properties, such as extra chromatic dispersion, a wide wavelength range single mode operation, and the tailorability of property *et al.*. The PCF is fabricated by introducing a defect or a missing hole in an array of regularly spaced tiny air holes along its propaga-

tion axis and the light is guided along the defect throughout the fiber length[1]. The adjustable properties of PCF suggest a lot of potential applications, including gas sensing, low transmission loss fiber, short-wavelength soliton transmission, dispersion compensation, and ultra broadband continuum generation. By designing novel structure and using different materials, the range of applications of the PCF should be expanded further[2,9].

As shown in Fig. 1, PCF can guide light using two different mechanisms. The first is based on photonic

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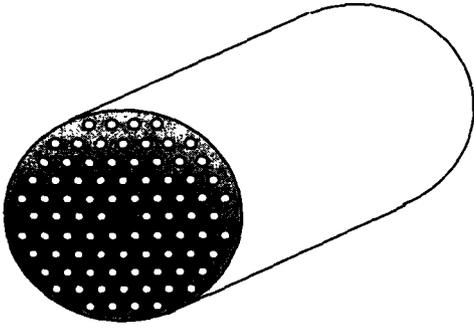


Fig. 1 Cross section of photonic crystal fiber where air holes are arranged in a hexagonal lattice in cladding region, and a defect is introduced
图1 光子晶体光纤的横截面空气孔以六边形分布在包层区域,结构中间引入了缺陷

band-gap effects, which is utilized to confine light in the core region. This type of wave-guide has been demonstrated experimentally. The second mechanism resembles that of conventional fiber due to the fact that the wave-guide is caused by a total internal reflection. The central defect in the PCF acts as the fiber core, and the surrounding periodic structure as cladding. Light is allowed to guide since index difference between the defect region and the air-silica microstructure cladding. In this paper the total internal reflection PCF is exclusively discussed, it doesn't depend on precise layout of air holes, but provides many interesting properties.

In this paper, we focus on the chromatic dispersion characteristics of PCF. In Sec. 1 we give the theoretic model and the effective index method in detail. The computing results and discussions will be presented in Sec. 2. Sec. 3 contains a summary.

1 Model and method

Let us assume that the PCF is uniform in the propagation direction (z). The transverse component of electric e_t satisfies the following vector wave equations [3],

$$\left\{ \nabla_t^2 + k_0^2 n^2 + \nabla_t \left[\left(\frac{\nabla_t n^2}{n^2} \right) \cdot \mathbf{o} \right] \right\} e_t = \beta^2 e_t, \quad (1)$$

where ∇_t denotes the gradient operator in the xy plane, $k_0 = 2\pi/\lambda$ is the free space wave number, and $n = n(x, y)$ is the refractive index profile and the vector

$e_t = (e_x, e_y)^T$ gives the transverse electric field.

Although the vector equation can be calculated accurately in principle via expanding the modal field and index profile by use of the Gaussian-Hermite function and the cosine (or sine) function as the set of basis functions, the calculation process can be quite complicated and time consuming, even prohibitive in practice. Fortunately, when PCF's air hole is small enough (which is also the condition of endless single mode), it resembles a weak wave guidance, and the scalar method provides useful approximation of the property of PCF. The effective index model introduced in reference [4,6] has been proved to be a significant scalar approach to approximate the dispersion and the mode field distribution of PCF. But the author didn't give the detailed process of this model. We here analyzed the effective index model in detail and present the eigenvalue equation of the cladding mode, and then use this model to discuss the dispersion characteristic of PCF.

In the effective index model, the effective index of cladding (air-silica structure) is computed by the propagation constant of the fundamental space-filling mode (FSM) of infinite air-silica configuration, β_{FSM} . The FSM is the fundamental mode in periodic air-hole silica structure without central defect, so the β_{FSM} is the maximum propagation constant permitted in the structure. The effective refractive index of cladding can be expressed as: $n_{eff} = \beta_{FSM}/k_0$, where k_0 is the free space wave number. Then, the effective V value of PCF can be obtained as:

$$V_{eff} = k_0 \Lambda (n_0^2 - n_{eff}^2)^{1/2}, \quad (2)$$

where Λ is the spacing of the air holes, which is used to denote the transverse dimension of the core of PCF,

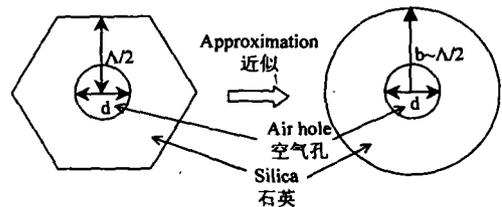


Fig. 2 Round approximation of hexagonal unit cell of photonic crystal fiber with central air hole
图2 光子晶体光纤的含空气孔六边形区域近似为圆形

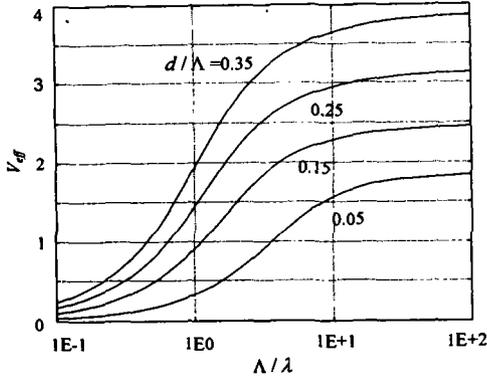


Fig. 3 Dependence of V_{eff} on Λ/λ with different air hole relative diameter d/Λ

图3 不同空气孔相对直径 d/Λ 时 V_{eff} 与 Λ/λ 的关系

n_0 is the refractive index of silica. The V value can be used to judge whether the fiber is single mode and to estimate the mode number supported in the fiber.

β_{FSM} is calculated in a scalar approximation. Because the infinite photonic crystal is periodical and symmetric, one can solve the FSM within a unit cell centered on one of the air holes with diameter d (Fig. 2). According to the symmetry of structure, the dege condition at the brim of unit can be expressed as $\partial\phi/\partial s = 0$, because of reflection symmetry, where ϕ is the scalar field, while s denote the direction normal to the edge [4]. Also, ϕ satisfies continuous condition at the air-silica interface. If the air hole is not too large, the unit cell can be approximate by a circle, so the edge condition is changed to $d\phi/dr = 0$ at $r = b$. The radius b of the circle can be deduced by equating the air hole fraction of round model to that of actual unit, or approximated to $\Lambda/2$ directly for a small air hole. This approximation is reasonable, since we mainly concerns the dispersion characteristics of PCF at endlessly single mode operation, which demands that the air-hole-diameter to the hole-hole pitch ratio d/Λ is less than a certain value.

Consider the above two edge conditions and the natural edge condition at the origin, the transverse scalar field component can be expressed as [11]:

$$\phi_r = \begin{cases} A I_0\left(\frac{W}{d/2}r\right), & r < d \\ B \left[J_0\left(\frac{U}{d/2}r\right) + C N_0\left(\frac{U}{d/2}r\right) \right], & d/2 < r < b \end{cases} \quad (3)$$

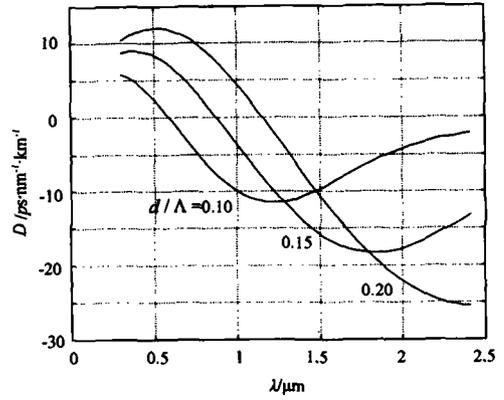


Fig. 4 Wave-guide dispersion of PCF for different d/Λ ratio at $\Lambda = 2.3\mu\text{m}$

图4 在 $\Lambda = 2.3\mu\text{m}$ 时,不同 d/Λ 比情况下的光子晶体光纤的波导色散

where $W = \sqrt{\beta_{FSM}^2 - n_1^2 k_0^2} \cdot d/2$, $U = \sqrt{n_0^2 k_0^2 - \beta_{FSM}^2} \cdot d/2$ (n_1 is the refractive index of air or any other material filled in the holes), $I_0(o)$, $J_0(o)$, $N_0(o)$ are the modified first kind of Bessel function of zero order, the first kind of Bessel function of zero order and the second kind of Bessel function of zero order, respectively.

Based on the relationship between transverse component and longitudinal component of the scalar field and the edge condition at $r = d/2$ and $r = \Lambda/2$, we reach the eigenvalue equation of the FSM in infinitely periodical air-silica structure as follow [11]

$$\frac{I_0(W) [J_0'(U) + C N_0'(U)]}{W} = \frac{I_0'(W) [J_0(U) + C N_0(U)]}{U}, \quad (4)$$

$$\text{where } C = -\frac{J_0'\left(\frac{U}{d/2}b\right)}{N_0'\left(\frac{U}{d/2}b\right)}$$

The maximum β value satisfying the eigenvalue equation is β_{FSM} .

2 Results and Discussions

One prominent aspect of PCF lies in that its waveguide property varies with the relative wavelength with respect to the air hole structure dimension Λ , but not the absolute wavelength. Hence, it is able to transfer one wavelength to another wavelength while keeping the waveguide dispersion unchanged by adjusting the dimension and spacing of the air holes the PCF. Depend-

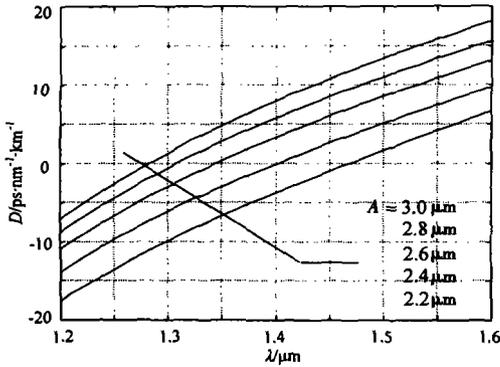


Fig. 5 Dependence of net dispersion (including wave-guide dispersion and material dispersion) on wavelength with respect to different air hole space Λ while d/Λ ratio is 0.15

图5 对于不同孔径 Λ 、 d/Λ 固定为0.15时,净色散(包括波导色散和材料色散)与波长的关系

ence of V_{eff} on Λ/λ with different air hole relative diameter d/Λ is shown in Fig. 3, where the material dispersion of silica is not considered in the case. The cutoff value V_{eff} for step-index fiber 2.405 cannot be directly applicable to the case of PCF since here the diameter of core is estimated, but it can be determined the value by experiment. For example, if the maximum d/Λ ratio is 0.2 that supports endlessly single mode operation, the cutoff V_{eff} value should be estimated to about 2.8 (corresponding curve is not plotted in the Fig. 3).

Having found β_{FSM} , it's not difficult to determine the dispersion characteristics of PCF. The method is not different from ordinary step index fiber. Fig. 4 illustrates the wave-guide dispersion of PCF with different air hole relative diameter d/Λ at air hole spacing $\Lambda = 2.3\mu\text{m}$. It is found that PCF can be anomalously ($D > 0$) dispersive under single mode operation, which is impossible for the step-index fiber. Therefore, it is able to shift PCF's zero dispersion point to less than $1.30\mu\text{m}$, which enables it to allow the possibility of soliton transmission at $1.3\mu\text{m}$ windows which need anomalous dispersion, and support dispersiveless transmission at shorter wavelength.

One of the most significant things about PCF is that its property can be tailored by adjusting the structure of PCF. It is much easier than the case of step index fiber, as in Fig. 4, which presents the variations of wave-guide dispersion of PCF with wavelength for different d/Λ ratio. As examples, Fig. 5 and Fig. 6 show the varia-

tions of total dispersion of PCF D for different d/Λ ratio, and the variations of total dispersion D for different air hole spacing Λ , respectively, where the material dispersion of glass is included and the refractive index is calculated by the Sellmeier formula with the parameters for the pure fused silica[10]. Fig. 5 shows that the zero total dispersion wavelength is decreased from $1.47\mu\text{m}$ to $1.28\mu\text{m}$ with the increase of Λ from $2.2\mu\text{m}$ to $3.0\mu\text{m}$, while d/Λ is 0.15. Fig. 6 also displays the large tunability of zero dispersion wavelength by adjusting the structure parameters of PCF.

For small air hole PCF, the wave-guide dispersion magnitude is limited as shown in Fig. 4 due to the small index difference between the core and the cladding. For large air hole PCF, the effective index model is invalid since the coupling between orthogonal field components can not be negligible. Large air hole will reduce the refractive index of the cladding, thus, the wave-guide dispersion is increased, the large air hole PCF has potential applications in dispersion compensation as described in [7]. Since the large air hole PCF does not support single mode operation at all wavelength range, it needs carefully design and chooses suitable wavelength band to work in single mode. And, the fiber core will be quite small to support single mode operation due to the large refractive index difference. The simple model of a solid silica rod surrounded by air can provide some insight into the large air hole PCF,

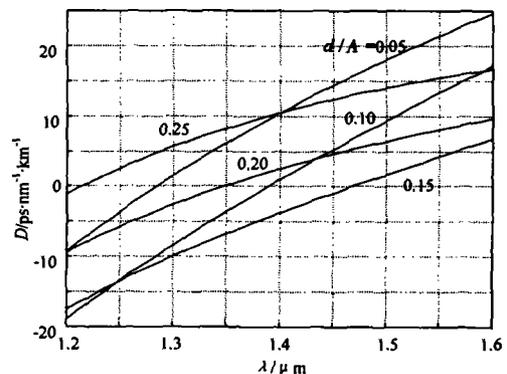


Fig. 6 Dependence of net dispersion (including wave-guide dispersion and material dispersion) on wavelength with respect to different d/Λ ratio while air hole space Λ is $2.2\mu\text{m}$

图6 对于不同 d/Λ 比、空气孔径 Λ 固定为 $2.2\mu\text{m}$ 时,净色散(包括波导色散和材料色散)与波长的关系

while the effective index model is invalid. To predict the modal property accurately, it is necessary to develop a precise model to solve the wave equation(s) in PCF, which is under considerations by the authors and many other researchers[3,5,8].

3 Conclusions

In summary, the chromatic dispersion properties of PCF have been discussed using the effective index model. We analyzed the refractive index model in detail, and presented the eigenvalue equation of FSM in the cladding. It has been shown that the wave-guide dispersion of PCF can be easily tailored by adjusting the structure parameters of PCF, and it is possible for PCF to be anomalously dispersive while under single mode operation, so we can shift the zero net dispersion (include material dispersion) below 1.3 μm , which provides the possibility for the short wavelength soliton transmission. The large air hole PCF and its applications in dispersion compensation are also discussed.

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