第18卷第1期
1999年2月

红外与毫米波学报
J. Infrared Millim. Waves

Vol. 18, No. 1
February, 1999

*

RIGOROUS, EFFICIENT, AND COMPLETE SOLUTION OF EIGENMODES IN MULTILAYER UNILATERAL FINLINES*

XU Yun-Sheng
(Department of Electronic Engineering and Information Science, University of Science and Technology of China, Hefei, Anhui 230027, China)

OMAR Abbas Sayed
(Arbeitsbereich Hochfrequenztechnik, Technische Universit t Hamburg-Harburg, D-21071 Hamburg, Germany)

Abstract Rigorous analysis of discontinuities in planar transmission lines may require accurate computation of a large number of modes. An improved formulation of the singular integral equation (SIE) method for multilayer unilateral finlines was presented to address this problem. All the series truncated possess the fast convergence property. A systematic approach for analytical calculation of the characteristic matrix to arbitrary order was also proposed. For the determination of propagation constants, an analytical function that eliminates all the poles in the determinant of the characteristic matrix was constructed. The developed numerical techniques lead to an accurate, efficient, and reliable computation of both propagation constants and field distributions for a large number of modes.

Key words eigenmodes, finlines, singular integral equation method.

Introduction

Rigorous characterization of discontinuities in planar passive circuits has been one of the most interesting research subjects. Among the various numerical techniques developed, the mode-matching method is frequently applied due to its advantageous features. However, an accurate analysis of strong discontinuities may require the determination of a large number of modes at both sides of a discontinuity. Hence, for a successful application of the mode-matching method, the numerical technique used for the solution of mode spec-

* The project supported by the Youth Science Foundation of the University of Science and Technology of China
Received 1998-10-05, revised 1998-12-22

tra should be: 1) accurate; 2) efficient, so that a large number of modes can be easily calculated; 3) complete or reliable, since missing of any intermediate mode solutions may eventually cause large errors in the mode-matching analysis of discontinuities . In view of the above three points, currently available techniques are not suitable for generating accurate, efficient, and complete solutions for a large number of modes. In this paper, we present an improved formulation of the singular integral equation (SIE) method which possesses the above-mentioned features for multilayer unilateral finlines.

The SIE method has been proved to be the most efficient and powerful method for the analysis of planar transmission lines[1~9]. For the calculation of a large number of modes in finlines, however, the existing formulations need further improvements. The main features of the present analysis are as follows: 1) Proper combinations of the tangential electric field or surface current components are used, based on which the series that the integral equations are finally derived from can be defined with fast convergence; 2) For the additionally imposed condition the series is accelerated by making use of its asymptotic behavior; 3) A systematic way for the analytical calculation of the characteristic matrix to arbitrary order is proposed, neither numerical integration nor summation of infinite series is necessary; 4) For the determination of propagation constants, an analytical function that eliminates all the poles in the determinant of the characteristic matrix is constructed. The developed numerical techniques lead to an accurate, efficient, and complete computation of both propagation constants and field distributions for a large number of modes.

1. The SIE Method for Multilayer Unilateral Finlines

The hybrid modes in a multilayer unilateral finline as shown in Fig. 1 can be treated as a superposition of LSE and LSM field parts. Each part may satisfy independently all the boundary conditions on the waveguide walls and the continuity requirements of the tangential field components at all strip-free interfaces. Only at the interface $x = 0$, the coupling between the two parts has to be taken into account so that the vanishing of the tangential electric field $\vec{E}_t$ on the metallic strips and of the surface current $\vec{J}_s$ in the slot can be guaranteed. $\vec{E}_t$ and $\vec{J}_s$ may be written as

$$\vec{E}_t = \vec{E}_t^h + \vec{E}_t^e, \quad \vec{J}_s = \vec{J}_s^h + \vec{J}_s^e \qquad (1)$$

where the superscripts h and e refer to the LSE and LSM parts, respectively. It is assumed that the variation of the modal fields along the longitudinal direction is described by $\exp(-j\beta z)$. $\vec{E}_t$ and $\vec{J}_s$ for the LSE part are completely characterized by their y-components,

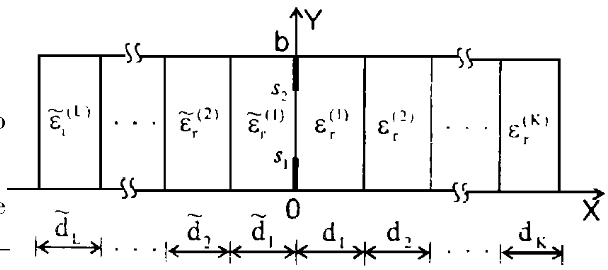

Fig. 1 Cross section of a multilayer unilateral finline.
图 1 多层鳍线的横截面示意图

whereas those for the LSM part can be derived from their z-components[3]. Thus, the whole problem can simply reduce to determining (E_y^h, J_y^h) for the LSE part and (E_z^e, J_z^e) for the LSM part at $x = 0$. They may be written as

$$E_y^h = \beta\sum_{n=0}^{\infty} A_n^h \cos(n\pi y/b), \quad J_y^h = \frac{\beta}{j\omega\mu_0}\sum_{n=0}^{\infty} F_n^h A_n^h \cos(n\pi y/b), \tag{2a}$$

$$E_z^e = j\beta\sum_{n=1}^{\infty} A_n^e \sin(n\pi y/b), \quad J_z^e = -\omega\varepsilon_0\beta\sum_{n=1}^{\infty} F_n^e A_n^e \sin(n\pi y/b), \tag{2b}$$

where F_n^h and F_n^e are the Fourier coefficients of Green's functions for LSE and LSM parts, respectively[9].

Now we formulate the SIE method for multilayer unilateral finlines. Two cosine series $f_1(y)$ and $f_2(y)$ are constructed in terms of the tangential electric field components, correspondingly two sine series $f_3(y)$ and $f_4(y)$ are constructed as a linear combination of the surface current components:

$$f_1(y) = -j\frac{dE_z}{dy} = \sum_{n=1}^{\infty} A_n^{(1)}\cos(\frac{n\pi}{b}y) \tag{3a}$$

$$f_2(y) = -E_y = \sum_{n=0}^{\infty} A_n^{(2)}\cos(\frac{n\pi}{b}y) \tag{3b}$$

$$f_3(y) = \left\{(K^e k_0^2 - K^h\beta^2)J_z - j\beta K^h\frac{dJ_y}{dy}\right\}/\omega\varepsilon_0 K^e K^h = \sum_{n=1}^{\infty} B_n^{(1)}\sin(\frac{n\pi}{b}y) \tag{3c}$$

$$f_4(y) = \left\{\beta J_z + j\frac{dJ_y}{dy}\right\}/\omega\varepsilon_0 K^e = \sum_{n=1}^{\infty} B_n^{(2)}\sin(\frac{n\pi}{b}y) \tag{3d}$$

where K^h and K^e are the limits of $(b/n\pi)F_n^h$ and $(b/n\pi)F_n^e$ for large n, respectively. Using (2) one can express $A_n^{(1)}$, $A_n^{(2)}$, $B_n^{(1)}$, and $B_n^{(2)}$ in terms of A_n^h and A_n^e.

Consider now two sine series defined by

$$g_i(y) = \sum_{n=1}^{\infty} A_n^{(i)}\sin(\frac{n\pi}{b}y) - f_{i+2}(y) = \sum_{n=1}^{\infty}\{A_n^{(i)} - B_n^{(i)}\}\sin(\frac{n\pi}{b}y), \quad i = 1, 2 \tag{4}$$

The carefully selected linear combinations in (3) make the series in (4) converge very fast and it can be proved that the asymptotic behavior of $\{A_n^{(i)} - B_n^{(i)}\}$ for large n is $n^{-5/2}$[9]. According to the continuity conditions at the interface $x = 0$, $f_1(y)$ and $f_2(y)$ should vanish on the strips. So, by using (3a) and (3b) one can express $A_n^{(i)}$ in terms of the functions $f_i(\varphi)$ defined in the slot $\varphi_1 \le \varphi \le \varphi_2$ only:

$$A_n^{(i)} = \frac{2}{\pi(1 + \delta_{n0})}\int_{\varphi_1}^{\varphi_2} f_i(y)\cos(n\varphi)\,d\varphi, \quad i = 1, 2, \tag{5}$$

with $\varphi = \pi y/b$, $\varphi_i = \pi s_i/b$. The continuity conditions at $x = 0$ also require vanishing $f_3(y)$ and $f_4(y)$ in the slot. Starting with the first equation in (4) in the slot, substituting (5) and 0 for $A_n^{(i)}$ and $f_{i+2}(y)$, respectively, summing the infinte series, and finally carrying out the transformation

$$\left.\begin{aligned}&\cos\varphi = Y_0 - X_0\eta,\\&Y_0 = \cos\{(\varphi_2+\varphi_1)/2\}\cos\{(\varphi_2-\varphi_1)/2\},\\&X_0 = \sin\{(\varphi_2+\varphi_1)/2\}\sin\{(\varphi_2-\varphi_1)/2\},\end{aligned}\right\} \tag{6}$$

one obtains the following standard singular integral equations:

$$-G_i(\eta) = \frac{1}{\pi}\int_{-1}^{1}\frac{F_i(\eta')}{\eta'-\eta}d\eta', |\eta| \le 1, \quad i = 1, 2 \tag{7}$$

with $G_i(\eta) = g_i(y)/\sin\varphi$, $F_i(\eta) = f_i(y)/\sin\varphi$. The analytical solutions for these equations are available. After replacing $G_i(\eta)$ by the second equation in (4), one can write $F_i(\eta)$ as

$$F_i(\eta) = \frac{1}{(1-\eta^2)^{1/2}}\{\frac{A_0^{(i)}}{X_0} + \sum_{n=1}^{\infty}(A_n^{(i)} - B_n^{(i)})q_n(\eta)\}, \quad A_0^{(i)} = 0, \quad i = 1, 2 \tag{8}$$

where $q_n(\eta)$ is defined by (A-1) in the appendix. Substitution of (8) into (5) yields

$$A_m^{(i)} = 2\tau_{m0}A_0^{(i)} + 2X_0\sum_{n=1}^{\infty}\sigma_{mn}\{A_n^{(i)} - B_n^{(i)}\}, \quad m \ge 1, \quad i = 1, 2 \tag{9}$$

where the integrals τ_{m0} and σ_{mn} are given by (A-2) and (A-3) in the appendix. For the case $m=0$, the preceding procedure results in an identity. By truncating the series in (9) after the N-th term and setting $m = 1, 2, \ldots, N$, we get $2N$ equations for $2N+1$ independent unknown coefficients. Examination of (3) reveals that the previously imposed conditions on $f_i(y)$ at the interface $x=0$ guarantee only constant E_z and J_y on the strips and in the slot, respectively. The vanishing of E_z at $y=0$ and $y=b$ makes such a constant for E_z automatically zero. For J_y, however, an additonal condition must be imposed to ensure its vanishing in the slot. Making use of the relation between (A_n^e, A_n^h) and $(B_n^{(1)}, B_n^{(2)})$, the series expression for J_y may be rewritten as

$$j\omega\mu_0 J_y = -F_0^h A_0^{(2)} + \sum_{n=1}^{\infty}\frac{K^h\beta B_n^{(1)} + (K^e k_0^2 - K^h\beta^2)B_n^{(2)}}{(n\pi/b)}\cos n\varphi \tag{10}$$

The asymptotic behavior of $B_n^{(1)}$ and $B_n^{(2)}$ for large n can be proved to be $n^{-1/2}$. Thus, the series in (10) converges more slowly than that in (9) and the direct use of (10) would slow down the overall speed of convergence. To accelerate its convergence, we express J_y as a sum of two series:

$$\begin{aligned}j\omega\mu_0 J_y = -F_0^h A_0^{(2)} &+ \sum_{n=1}^{\infty}\frac{K^h\beta(B_n^{(1)} - A_n^{(1)}) + (K^e k_0^2 - K^h\beta^2)(B_n^{(2)} - A_n^{(2)})}{(n\pi/b)}\cos n\varphi\\&+ \sum_{n=1}^{\infty}\frac{K^h\beta A_n^{(1)} + (K^e k_0^2 - K^h\beta^2)A_n^{(2)}}{(n\pi/b)}\cos n\varphi\end{aligned} \tag{11}$$

Substituting (5) into the second series of (11), summing it according to [10]

$$\sum_{n=1}^{\infty}\frac{\cos(n\varphi)\cos(n\varphi')}{n} = -\frac{1}{2}\ln\{2|\cos\varphi - \cos\varphi'|\}, \quad 0 \le \varphi, \varphi' \le \pi, \tag{12}$$

one obtains

$$j\omega\mu_0 J_y = -F_0^h A_0^{(2)} + \sum_{n=1}^{\infty} \frac{K^h\beta(B_n^{(1)} - A_n^{(1)}) + (K^e k_0^2 - K^h\beta^2)(B_n^{(2)} - A_n^{(2)})}{(n\pi/b)} \cos n\varphi$$

$$-\frac{X_0 b}{\pi^2}\int_{-1}^{1}\{K^h\beta F_1(\eta') + (K^e k_0^2 - K^h\beta^2)F_2(\eta')\}\ln(2X_0|\eta' - \eta|)d\eta' \qquad (13)$$

In principle one can now impose zero J_y at any point in the slot. In order to avoid here the appearance of improper integrals so as to simplify integration, however, we let the sum of the values of J_y at $y = s_1$ and $y = s_2$ be zero instead. After replacing $F_1(\eta)$ and $F_1(\eta)$ by (8), the imposed additional condition for J_y is then written as

$$-\left\{2F_0^h + \frac{b}{\pi}(K^e k_0^2 - K^h\beta^2)\zeta_0\right\}A_0^{(2)} + \frac{b}{\pi}\sum_{n=1}^{\infty}\{[K^h\beta(B_n^{(1)} - A_n^{(1)}) +$$

$$(K^e k_0^2 - K^h\beta^2)(B_n^{(1)} - A_n^{(1)})]\cdot[X_0\xi_n + (\cos n\varphi_1 + \cos n\varphi_2)/n]\} = 0 \qquad (14)$$

where ζ_0 and ξ_n are integrals given by (A-4) and (A-5) in the appendix, respectively. As $A_n^{(i)} - B_n^{(i)}$ rapidly approaches zero, the convergence is accelerated.

Through (9) and (14) truncated behind the N^{th} term, one gets a system of characteristic equations:

$$[C]\cdot[X] = 0 \qquad (15)$$

where [C] is the characteristic matrix of order $2N+1$, and [X] is a column vector composed of $2N+1$ independent unknown coefficients (A_n^e, A_n^h). As can be seen from the preceding procedures, all the series truncated in (15) converge rapidly with the order of $n^{-5/2}$ to zero. Thus a characteristic matrix of a relatively small order in comparison with other methods can be used to determine a large number of modes accurately and efficiently.

Equation (15) is a nonstandard matrix eigenvalue problem. It can only be solved by regarding the determinant of the characteristic matrix [C] as a function in the eigenvalue to be determined and looking for its zeros. The calculation of the determinant should be efficient, especially when a large number of modes need to be determined. The fast convergence property of the series truncated has laid a good foundation for an efficient computation. The remaining problem is the accurate and efficient computation of various integrals contained in the elements of [C]. An analytical approach for the computation of these integrals in [C] to arbitrary order is described in the appendix. Neither numerical integration nor summation of infinite series is necessary.

The determinant of [C] contains poles in addition to the zeros to be searched for. Some of the poles are located very close to zeros, so they may greatly interfere the root-finding process. As a consequence, some zeros may be missing from the mode spectrum. We construct an analytical function that contains exactly the same set of zeros as in the determinant of [C], but eliminates all its poles. In this way, the complete determination of propagation constants can be ensured. We will discuss this topic in detail in a subsequent publication.

2. Results and Discussions

If a finline is symmetric with respect to $y=b/2$, the modes can then be classified as odd (E_z-odd, H_z-even) or even (E_z-even, H_z-odd) modes and the characteristic matrix can be decomposed into two decoupled submatrices.

Table 1 Convergence of propagation constants β/k_0 for odd modes in a finline with varying order of the characteristic matrix. Parameters: $K=1$, $L=2$, $f=35$GHz, $\tilde{\epsilon}_r^{(1)}=2.22$, $\tilde{\epsilon}_r^{(2)}=\epsilon_r^{(1)}=1.0$, $\tilde{d}_2=d_1=3.429$, $\tilde{d}_1=0.254$, $s_1=1.278$, $s_2=2.278$, $b=3.556$mm.

表1 一鳍线中奇模归一化传播常数 β/k_0 随 N 增大时的收敛特性. $K=1$, $L=2$, $f=35$/ GHz, $\tilde{\epsilon}_r^{(1)}=2.22$, $\tilde{\epsilon}_r^{(2)}=\epsilon_r^{(1)}=1.0$, $\tilde{d}_2=d_1=3.429$, $\tilde{d}_1=0.254$, $s_1=1.278$, $s_2=2.278$, $b=3.556$mm

N	β_1/k_0	β_2/k_0	β_3/k_0	β_4/k_0	β_5/k_0	β_6/k_0	β_7/k_0	β_8/k_0 ($\beta_9=-\beta_8^*$)	β_{10}/k_0
2	0.997666	- j0.662224	- j1.03783	- j2.12437	- j2.18834	- j2.23986	- j2.35776	0.01068- j2.49830	- j2.52318
4	0.995294	- j0.662247	- j1.03783	- j2.12438	- j2.18834	- j2.23958	- j2.35764	0.01069- j2.49824	- j2.52313
6	0.995270	- j0.662247	- j1.03782	- j2.12434	- j2.18834	- j2.23957	- j2.35759	0.01069- j2.49822	- j2.52309
8	0.995121	- j0.662247	- j1.03777	- j2.12431	- j2.18834	- j2.23957	- j2.35752	0.01069- j2.49821	- j2.52307
10	0.995084	- j0.662246	- j1.03773	- j2.12430	- j2.18834	- j2.23957	- j2.35750	0.01069- j2.49821	- j2.52307
12	0.995083	- j0.662246	- j1.03773	- j2.12430	- j2.18834	- j2.23957	- j2.35750	0.01069- j2.49820	- j2.52307
14	0.995083	- j0.662246	- j1.03773	- j2.12430	- j2.18834	- j2.23957	- j2.35750	0.01069- j2.49820	- j2.52307
20	0.995084	- j0.662245	- j1.03772	- j2.12430	- j2.18834	- j2.33957	- j2.35749	0.01069- j2.49820	- j2.52307
30	0.995085	- j0.662245	- j1.03771	- j2.12430	- j2.18834	- j2.23957	- j2.35749	0.01069- j2.49820	- j2.52307
2	0.9972	- j0.6608	- j1.0373	- j2.1228		- j2.2380	------ [8]		

N	β_{20}/k_0	β_{30}/k_0	β_{40}/k_0	β_{50}/k_0
2	- j4.33821	- j6.61264	- j8.53876	- j10.5187
4	- j4.33741	- j5.23067	- 0.01179- j5.91982	- j6.85409
6	- j4.33680	0.00295- j5.23078	- 0.00894- j5.91529	- j6.84814
8	- j4.33654	0.00367- j5.22905	- 0.00561- j5.91328	- j6.84516
10	- j4.33649	0.00375- j5.22867	- 0.00447- j5.91281	- j6.84445
12	- j4.33649	0.00375- j5.22867	- 0.00448- j5.91280	- j6.84443
14	- j4.33648	0.00377- j5.22860	- 0.00431- j5.91274	- j6.84438
20	- j4.33647	0.00378- j5.22852	- 0.00406- j5.91266	- j6.84427
30	- j4.33646	0.00379- j5.22848	- 0.00396- j5.91263	- j6.84424

N	β_{60}/k_0	β_{70}/k_0	β_{80}/k_0	β_{90}/k_0	β_{100}/k_0
2	- j12.6655	- j14.3314	- j16.4064	- j18.7307	- j20.2805
4	- j7.83642	- j9.00216	- j10.1844	- j11.2846	- j12.5054
6	- j7.37428	- j7.83579	- j8.55968	- j9.14815	- j9.95070
8	- j7.37433	- j7.83382	- j8.55349	- j9.14794	0.00337- j9.66378
10	- j7.37433	- j7.83361	- j8.55238	- j9.14764	0.00339- j9.66370
12	- j7.37433	- j7.83361	- j8.55236	- j9.14760	0.00339- j9.66368
14	- j7.37433	- j7.83355	- j8.55227	- j9.14760	0.00339- j9.66368
20	- j7.37433	- j7.83349	- j8.55211	- j9.14755	0.00339- j9.66368
30	- j7.37433	- j7.83347	- j8.55205	- j9.14754	0.00340- j9.66368

Table 2 Convergence of propagation constants β/k_0 for even modes in a finline with varying order of the characteristic matrix (see Table 1 for parameters)

表 2　一鳍线中偶模归一化传播常数 β/k_0 随 N 增大时的收敛特性(几何及电参数同表 1)

N	β_1/k_0	β_2/k_0	$\beta_3/k_0(\beta_4=-\beta_3^*)$	β_5/k_0	β_6/k_0	β_7/k_0	β_8/k_0	β_9/k_0	β_{10}/k_0
1	- j0.652830	- j0.712487	0.01918- j1.33942	- j1.40271	- j1.45848	- j2.40772	- j2.42135	- j2.55783	- j2.60946
3	- j0.652861	- j0.712608	0.01685- j1.33671	- j1.40184	- j1.45651	- j2.37402	- j2.42696	- j2.55490	- j2.60507
5	- j0.652873	- j0.712674	0.01602- j1.33593	- j1.40160	- j1.45614	- j2.36511	- j2.42747	- j2.55412	- j2.60459
7	- j0.652873	- j0.712672	0.01593- j1.33584	- j1.40157	- j1.45609	- j2.36402	- j2.42752	- j2.55403	- j2.60454
9	- j0.652873	- j0.712672	0.01593- j1.33584	- j1.40157	- j1.45609	- j2.36402	- j2.42752	- j2.55403	- j2.60454
11	- j0.652873	- j0.712666	0.01592- j1.33583	- j1.40157	- j1.45608	- j2.36388	- j2.42752	- j2.55402	- j2.60453
13	- j0.652872	- j0.712661	0.01592- j1.33582	- j1.40157	- j1.45607	- j2.36378	- j2.42752	- j2.55401	- j2.60452
19	- j0.652872	- j0.712659	0.01592- j1.33581	- j1.40157	- j1.45606	- j2.36374	- j2.42752	- j2.55401	- j2.60451
29	- j0.652872	- j0.712658	0.01592- j1.33581	- j1.40157	- j1.45606	- j2.36372	- j2.42752	- j2.55401	- j2.60451
3	- j0.6512	- j0.7086		- j1.4022	- j1.4536	------ [8]			

N	β_{20}/k_0	β_{30}/k_0	β_{40}/k_0	β_{50}/k_0	β_{60}/k_0	β_{70}/k_0	β_{80}/k_0	β_{90}/k_0	β_{100}/k_0
1	- j5.86082	- j8.78078	- j11.6559	- j15.0167	- j17.5140	- j20.9571	- j23.7588	- j26.7726	- j30.0056
3	- j3.86587	- j5.05299	- j6.30842	- j7.85830	- j9.33413	- j10.6397	- j12.1880	- j13.7713	- j15.1445
5	- j3.85697	- j5.05161	- 0.00524- j6.06046	- j6.49755	- j7.30293	- j8.17514	- j8.95569	- j9.96917	- j10.7562
7	- j3.85620	- j5.05156	- 0.00523- j6.06034	- j6.49409	- j7.28318	- j8.17415	- j8.53833	- j8.95200	- j9.47131
9	- j3.85620	- j5.05156	- 0.00522- j6.06030	- j6.49397	- j7.28317	- j8.17409	- j8.53609	- j8.95179	- j9.47131
11	- j3.85607	- j5.05153	- 0.00522- j6.06030	- j6.49389	- j7.28232	- j8.17397	- j8.53593	- j8.95110	- j9.47062
13	- j3.85599	- j5.05152	- 0.00522- j6.06029	- j6.49377	- j7.28168	- j8.17392	- j8.53593	- j8.95054	- j9.47024
19	- j3.85595	- j5.05151	- 0.00522- j6.06029	- j6.49372	- j7.28146	- j8.17390	- j8.53591	- j8.95035	- j9.47010
29	- j3.85593	- j5.05151	- 0.00522- j6.06029	- j6.49370	- j7.28135	- j8.17390	- j8.53591	- j8.95026	- j9.47003

Tables 1 and 2 show the convergence of odd and even modes in a finline with respect to the series truncation order N, respectively. The results of [8] using the SIE method for the "first" nine modes are also given and they agree well with our results. However, the reliability of the computation in [8] seems to be seriously in question. The fifth odd mode and the third and fourth even complex ones are missing from the mode spectrum. Within the first 100 odd modes, those with the following mode numbers are complex: 8, 9, 12, 13, 18, 19, 25, 26, 30—33, 39, 40, 56, 57, 61—64, 81, 82, 100; the even complex modes are numbered as follows: 3, 4, 15, 16, 24, 25, 39, 40, 47, 48, 54, 55, 73, 74, 77, 78, 84, 85, 92, 93. As can be seen from the Tables, a characteristic matrix of order 11×11 ($N=10$) is sufficient for the rigorous computation of the first 100 odd modes within an error of 0.01%, whilst a matrix of order 14×14 ($N=13$) gives accurate results for the first 100 even modes within an error of 0.005%. Figures 2 and 3 show the calculated current components J_y and J_z at the interface $x=0$ for the dominant mode and the 100th odd complex mode, respectively. They satisfy very well the continuity requirements and J_z shows the correct singularity behavior. Excellent results can also be obtained for the electric field components E_y and E_z at

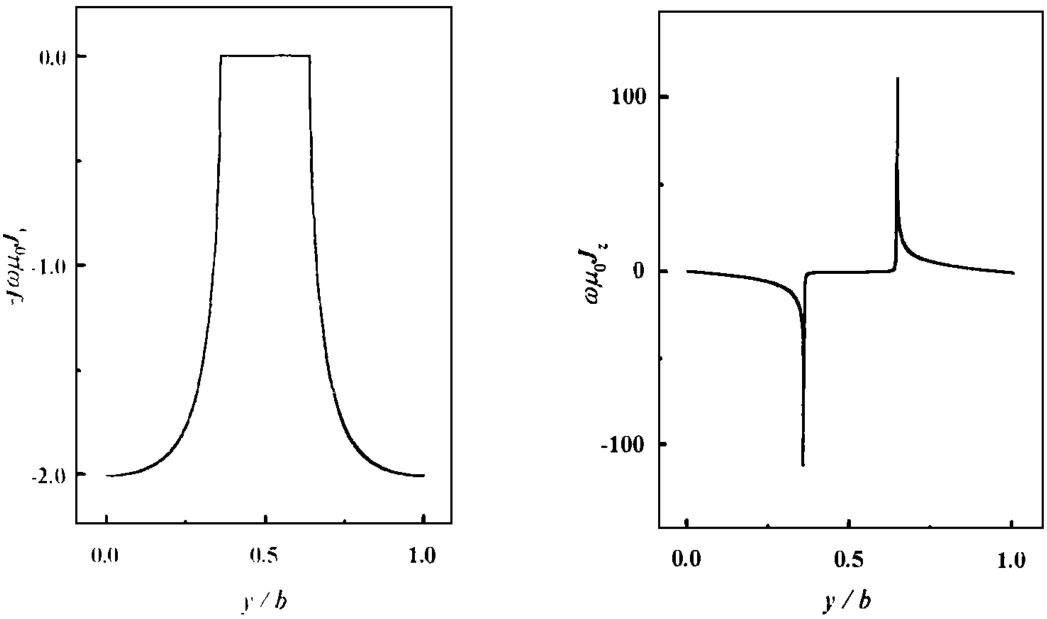

Fig. 2 Current distributions of the dominant mode at the interface $x=0$
(see Table 1 for parameters)

图2 在$x=0$的交界处主模(第一个奇模)的面电流密度(几何及电参数同表1)

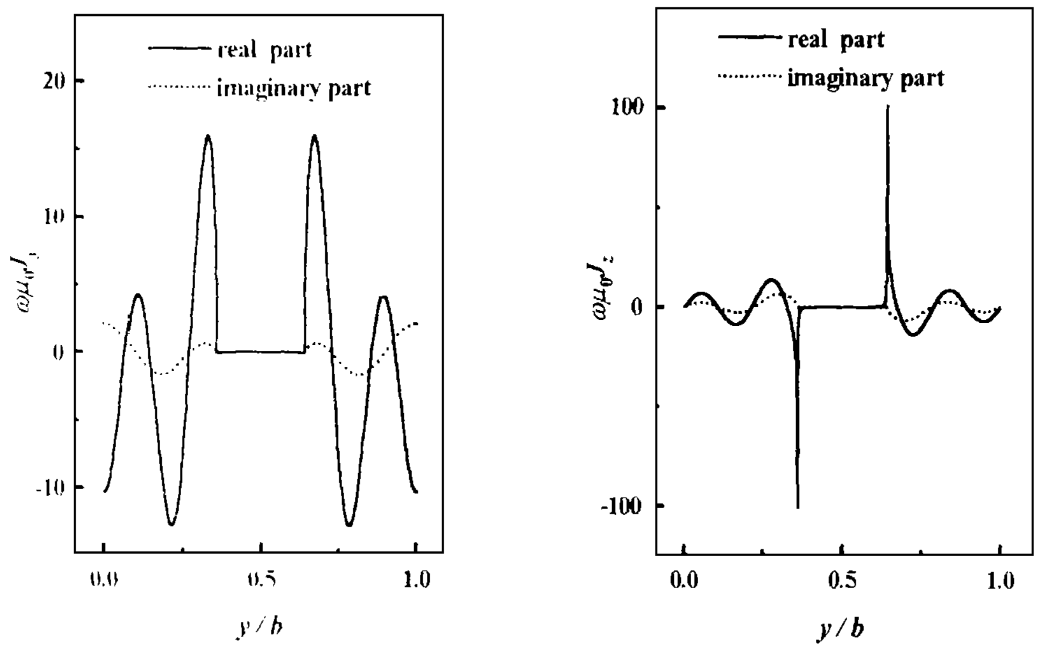

Fig. 3 Current distributions of the 100th odd complex mode at the interface $x=0$
(see Table 1 for parameters)

图3 在$x=0$的交界处第100个复数奇模的面电流密度(几何及电参数同表1)

$x = 0$. The calculated field distributions given in [8] do not contain any singularity behaviors and hence are inaccurate.

3. Conclusions

An improved formulation of the singular integral equation method for multilayer unilateral finlines is presented. The developed techniques lead to an accurate, efficient and complete solution of a large number of modes in finlines for the first time. This lays a good foundation for rigorous analysis of discontinuities using the mode-matching method.

Appendix

In the formulation of the SIE method, the following integrals are concerned and can be solved analytically or evaluated with a recurrence relation:

$$q_n(\eta) = \frac{1}{\pi}\int_{-1}^{1} \frac{(1-\eta'^2)^{1/2}}{\eta'-\eta}\frac{\sin n\varphi'}{\sin\varphi'}d\eta', \quad |\eta| < 1 \tag{A-1}$$

$$\tau_{mn} = \frac{1}{\pi}\int_{-1}^{1}\cos m\varphi\frac{\eta^n}{(1-\eta^2)^{1/2}}d\eta \tag{A-2}$$

$$\sigma_{mn} = \frac{1}{\pi}\int_{-1}^{1}\cos m\varphi\frac{q_n(\eta)}{(1-\eta^2)^{1/2}}d\eta \tag{A-3}$$

$$\zeta_n = \frac{1}{\pi}\int_{-1}^{1}\frac{\eta^n\ln\{4X_0^2(1-\eta^2)\}}{(1-\eta^2)^{1/2}}d\eta \tag{A-4}$$

$$\xi_n = \frac{1}{\pi}\int_{-1}^{1}\frac{q_n(\eta)\ln\{4X_0^2(1-\eta^2)\}}{(1-\eta^2)^{1/2}}d\eta \tag{A-5}$$

The analytical expressions for the integrals given below are needed for the evaluation of the above integrals[9]:

$$C_n = \frac{1}{\pi}\int_{-1}^{1}\frac{\eta^n d\eta}{(1-\eta^2)^{1/2}} = \begin{cases} 0, & n = 1,3,5,\ldots \\ (2k)!/(2^{2k}k!k!), & n = 2k = 0,2,4,\ldots \end{cases} \tag{A-6}$$

$$D_n = \frac{1}{\pi}\int_{-1}^{1}\eta^n(1-\eta^2)^{1/2}d\eta = C_n/(n+2), \tag{A-7}$$

$$Q_n(\eta) = \frac{1}{\pi}\int_{-1}^{1}\frac{(1-\eta'^2)^{1/2}\eta'^n}{(\eta'-\eta)}d\eta' = -\eta^{n+1} + \sum_{k=0}^{[\frac{n-1}{2}]}D_{2k}\eta^{n-1-2k}, \quad |\eta| \le 1 \tag{A-8}$$

where $[(n-1)/2]$ represents the integer part of $(n-1)/2$ and if $n = 0$, the result of the summation in (A-8) is defined as zero.

$\sin(n\varphi)/\sin\varphi$ in (A-1) is a polynomial in η since it may be written as

$$\sin(n\varphi)/\sin\varphi = U_{n-1}(\cos\varphi) = U_{n-1}(Y_0 - X_0\eta) \tag{A-9}$$

where $U_n(x)$ is the Chebychev polynomial of the second kind. Applying (A-9) and (A-8) to (A-1), the analytical expression for $q_n(\eta)$ can be obtained as a polynominal in η. For this reason, if (A-2) and (A-4) are available analytically, (A-3) and (A-5) can also be solved.

Cos$m\varphi$ can be written as a polynomial in η through the relation

$$\cos(m\varphi) = T_m(\cos\varphi) = T_m(Y_0 - X_0\eta) \qquad (A-10)$$

where $T_m(x)$ is the Chebychev polynomial of the first kind. Substituting the polynomial expression of cos$m\varphi$ into (A-2), τ_{mn} can be integrated using (A-6).

For the following type of integral

$$I_n = \int_{\eta_1}^{\eta_2} f(\eta)\frac{\eta^n}{\sqrt{R(\eta)}}d\eta$$

$$(R(\eta) = a_0\eta^2 + a_1\eta + a_2, \quad a_0 \neq 0) \qquad (A-11)$$

where $f(\eta)$ is any derivable function, the recurrence relation can be obtained as follows:

$$I_n = \frac{1}{2na_0}\{2f(\eta)\eta^{n-1}\sqrt{R(\eta)}\Big|_{\eta_1}^{\eta_2} - 2\int_{\eta_1}^{\eta_2}\eta^{n-1}f'(\eta)\sqrt{R(\eta)}d\eta - (2n-1)a_1I_{n-1}$$

$$-(2n-2)a_2I_{n-2}\} \qquad (A-12)$$

For $n=1$ the above formula is still valid by setting the last term to zero. So if the second term in the bracket of (A-12) is available in closed form, it is only necessary to solve the first integral I_0 for the integrals as in (A-11). Applying (A-12) to (A-4), one obtains the following recurrence formula:

$$\zeta_0 = \ln(X_0^2), \quad \zeta_n = \frac{1}{n}\{-2C_n + (n-1)\zeta_{n-2}\} \qquad (A-13)$$

REFERENCES

1 Mittra R, Itoh T. *IEEE Trans. Microwave Theory Tech.*, 1971, **19**: 47～56

2 Omar A S, Schünemann K. *IEEE Trans. Microwave Theory Tech.*, 1984, **32**(12): 1626～1632

3 Omar A S, Schünemann K. *IEEE Trans. Microwave Theory Tech.*, 1985, **33**(12): 1313～1322

4 Huang W X, Itoh T. *IEEE Trans. Microwave Theory Tech.*, 1988, **36**(1): 163～165

5 Uzunoglu N K, Capsalis C N, Chronopoulos C P. *IEEE Trans. Microwave Theory Tech.*, 1988, **36**(6): 976～984

6 Uhde K. *Theorie und Realisierung von optoelektronischen Schaltern in Finleitungstechnik, Ph. D. dissertation. Technische Universit t Hamburg-Harburg, Germany*, 1990

7 Huang Y-F, Lai S-L. *IEE Proc. (pt. H)*, 1993, **140**(1): 11～16

8 Huang Y-F, Lai S-L. *IEEE Trans. Microwave Theory Tech.*, 1994, **42**(1): 84～91

9 Xu Y-S, Omar A S. *IEEE Trans. Microwave Theory Tech.*, 1994, **42**(7): 1213～1222

10 R. E. Collin. *Field Theory of Guided Waves*, 2nd ed.. New York: IEEE Press, 1991

*

多层鳍线本征模的精确有效和完备求解*

徐云生
(中国科学技术大学电子工程与信息科学系, 安徽, 合肥, 230027)

Omar Abbas Sayed
(Arbeitsbereich Hochfrequenztechnik, Technische Universit t Hamburg-Harburg, D-21071 Hamburg, Germany)

摘要　对多层鳍线现有的奇异积分方程法进行了如下改进: 1) 适当选择导体带所在平面切向电场和表面电流的线性组合, 以使得导出奇异积分方程的级数具有很快的收敛特性; 2) 对于附加的边界条件, 利用其级数的渐近特性来加速其收敛; 3) 给出系统计算任意阶特征矩阵元素的解析方法, 以避免数值积分或对无穷级数求和; 4) 为完备地求解本征模的传播常数, 构造了一个解析函数, 保留特征矩阵行列式的所有零点, 但消除其所有奇点. 采用本文的奇异积分方程法, 首次精确有效完备地求解了鳍线中的大量本征模.

关键词　本征模, 鳍线, 奇异积分方程法.

* 中国科学技术大学青年科学基金、中国科学院留学经费择优支持回国工作基金以及国家教育委员会留学回国人员科研资助费资助项目
稿件收到日期1998-10-05, 修改稿收到日期1998-12-22