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Accurate atmospheric transmittance model for O_2 absorption band near 762 nm

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Abstract: An atmospheric transmittance model for molecular oxygen absorption band near 762 nm is expressed by a polynomial including temperature and pressure. The polynomial coefficients are expressed as another power series of cosine of incident angle. Similarly, a mean atmospheric transmittance model is also given. Numerical tests indicate that under vast majority of circumstances the fitting errors (RMS) of the atmospheric transmittance model and those of the mean atmospheric transmittance model are less than 0.0001 and 0.0005, and their maximal errors are less than 0.0005 and 0.003, respectively. Both models were applied to the study on remote sounding of atmospheric pressure profile from space. In addition, a correction method is given for the interval error of atmospheric transmittance.

Key words: atmospheric transmittance; atmospheric absorption; O_2 absorption band; transmittance model; atmospheric remote sounding
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762 nm 氧分子吸收带在非均匀大气中不同投射角的大气透过率准确模型

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摘要:762 nm 氧分子吸收带的大气透过率模型用包含温度和气压的多项式表示,其多项式系数表示为入射角的余弦的另一个幂级数.类似地,还给出了平均大气透过率模型.数值试验表明,在绝大多数情况下,透过率模型与平均透过率模型的拟合误差(RMS)分别小于0.0001和0.0005,其极大误差分别小于0.0005和0.003.此二模型已用于从空间探测气压廓线的研究.此外,还给出了大气透过率间隔误差的订正方法.

关 键 词:大气透过率;大气吸收;O_2吸收带;透过率模型;大气探测
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Introduction

Atmospheric remote sounding from space is based on spectral radiometry and calculations of atmospheric transmittances. Therefore, it is very essential to establish an accurate and computationally fast transmittance model. In 1969 Smith has given polynomial representations of carbon dioxide and water vapor transmittances[1], which were applied to study on pressure sensing of the stratosphere and the lower

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mesosphere by solar occultation[2] and to study on remote sounding of the mixing ratio of carbon dioxide in the atmosphere from space[3]. In 1970s and 1990s, McMillin, Fleming, et al. have provided different transmittance models of atmospheric absorbing gases[4-8]. These models were applied to data processing of TOVS on the meteorological satellites NOAA in the United States[9]. Later, Eyre has also given a computationally fast transmittance model[10], which was applied by the United Kingdom Meteorological Office (UKMO) to calculation of the TOVS atmospheric transmittance. On the other hand, an atmospheric transmittance model was established also for the molecular oxygen absorption band near 762 nm with lower accuracy[11].

Based on physical essence, we would like to give a very accurate, computationally fast and convenient atmospheric transmittance model and a mean atmospheric transmittance model in an inhomogeneous atmosphere at different incident angles for the O_2 absorption band near 762 nm. For establishing the models the atmospheric refraction and the global curvature are taken into account. In this study we regarded the atmospheric pressures at different altitudes in the U. S. Standard Atmosphere 1976 (USSA)[12], $\tilde{p}$, as an altitude scale and divide the atmosphere into 40 levels or layers as same as specified by NOAA / NESDIS of the United States[4]. The lowest pressure level of the atmospheric pressure profile of the USSA is 1000 hPa corresponds to an altitude of 110.26 m. Consequently, both the models given in this paper can be applied to the atmospheric pressure range at the altitude of 110.26 m from 1075 hPa to 925 hPa. In order to calculate the atmospheric transmittance from the top of the atmosphere to the earth surface, τ_s, an extrapolation is needed by using an available atmospheric transmittance profile. The atmospheric transmittance model and the mean atmospheric transmittance model given in this paper were applied to the study on the remote sounding of atmospheric pressure profile from space[13-15].

1 Theoretical basis

The atmosphere is divided into n layers. Between the top of the atmosphere ($i=0$) and the first pressure level ($i=1$) is the first layer ($i=1$). The upper and the lower boundaries of the i^{th} layer are defined by the $(i-1)^{\text{th}}$ and the i^{th} pressure levels, respectively. Then, the monochromatic atmospheric transmittance τ_i of molecular oxygen in a slant path from the top of the atmosphere to the i^{th} level can be expressed as a product of the monochromatic atmospheric transmittance τ_{i-1} from the top of the atmosphere to the $(i-1)^{\text{th}}$ level and the monochromatic atmospheric transmittance of the i^{th} layer, $\Delta\tau_i$, namely,

$$\tau_i = \tau_{i-1}\Delta\tau_i \quad . \qquad (1)$$

Considering the equation of state gives the expression of $\Delta\tau_i$ in equation (1):

$$\begin{aligned}\Delta\tau_i &= \exp(-L_{O_2}) = \exp\left(-\int_{z_i}^{z_{i-1}} k_i q_{O_2}\rho_i\mu_i \mathrm{d}z\right) \\ &\approx \exp(-k'_i\rho'_i\mu'_i q_{O_2}\Delta z_i) \\ &= \exp\left(-k'_i\frac{Mp'_i\mu'_i q_{O_2}\Delta z_i}{R^*T'_i}\right)\end{aligned} \quad , \qquad (2)$$

in which L, k, q_{O_2}, ρ, z, μ, M, R^* and T are optical depth, absorption coefficient, O_2 mass mixing ratio with a value of 0.209476, atmospheric density, altitude, cosine of incident angle θ, molecular weight of dry air, universal gas constant and temperature, respectively; furthermore, prime represents mean values in the the i^{th} layer and generally, averages in a layer, e.g., $p'_i = (p_{i-1}+p_i)/2$.

The absorption coefficient k is a product of spectral line strength S and spectral line pattern function G. In the O_2 absorption band near 0.762 μm, both S and G are functions of $1/T$ and p[16], herein G may be the Lorentz spectral line pattern function G_L (for lower atmosphere), or the Doppler spectral line pattern function G_D (for upper atmosphere), or the mixing spectral line pattern function G_m (for whole atmosphere). On the other hand, the O_2 mixing ratio q_{O_2} is constant below about 80 km. Therefore, letting the constant

$$\gamma_i = \frac{M\mu'_i q_{O_2}\Delta z_i}{R^*} \quad , \qquad (3)$$

Eq. (2) can be rewritten as

$$\Delta\tau_i = \exp\left(-\gamma'_i k'_i \frac{p'_i}{T'_i}\right) \quad . \qquad (4)$$

Consequently, the function in square bracket in Eq. (4) can be expanded as a power series:

$$\Delta\tau_i = 1 - \frac{1}{1!}\gamma'_i k'_i\frac{p'_i}{T'_i} + \frac{1}{2!}\left(\gamma'_i k'_i\frac{p'_i}{T'_i}\right)^2 - \cdots \quad . \qquad (5)$$

All the terms in the square bracket in Eq. (5) can be regarded as a functions of $1/T_i$, $1/T_{i-1}$ and $1/T'_i$ as well as p_i, p_{i-1} and p'_i. Further, we can expand $\Delta\tau_i$ as a new polynomial that can be expressed as a sum of first order terms, second order terms, couple terms of

$1/T_i$, $1/T_{i-1}$ and $1/T_i'$ as well as p_i, p_{i-1} and p_i', and a remaining term $O(1/T_{i-1}, 1/T_i, 1/T_i', p_{i-1}, p_i, p_i')$ that is very small and can be omitted:

$$\begin{aligned}\Delta\tau_i = &\beta_{i,1} + \beta_{i,2}\frac{1}{T_{i-1}} + \beta_{i,3}\frac{1}{T_i} + \beta_{i,4}\frac{1}{T_i'} + \beta_{i,5}\frac{1}{T_{i-1}^2} \\ &+ \beta_{i,6}\frac{1}{T_i^2} + \beta_{i,7}\frac{1}{T_i'^2} + \beta_{i,8}p_{i-1} + \beta_{i,9}p_i + \beta_{i,10}p_i' \\ &+ \beta_{i,11}p_{i-1}^2 + \beta_{i,12}p_i^2 + \beta_{i,13}p_i'^2 + \beta_{i,14}\frac{p_{i-1}}{T_{i-1}} \\ &+ \beta_{i,15}\frac{p_i}{T_{i-1}} + \beta_{i,16}\frac{p_i'}{T_{i-1}} + \beta_{i,17}\frac{p_{i-1}}{T_i} + \beta_{i,18}\frac{p_i}{T_i} \\ &+ \beta_{i,19}\frac{p_i'}{T_i} + \beta_{i,20}\frac{p_{i-1}}{T_i'} + \beta_{i,21}\frac{p_i}{T_i'} + \beta_{i,22}\frac{p_i'}{T_i'} \\ &+ O\left(\frac{1}{T_{i-1}}, \frac{1}{T_i}, \frac{1}{T_i'}, p_{i-1}, p_i, p_i'\right) \quad . \end{aligned} \tag{6}$$

2 Numerical models

Any radiometric instrument has a bandwidth Δv, so that all transmittances needed are always normalized polychromatic transmittances in the channels with a bandwidth of Δv, i. e. $\tau_i(\Delta v)$. They are concerned with the atmospheric temperature profile and the atmospheric pressure profile over the i^{th} level. Therefore, some bandwidth modification terms similar to those provided by McMillin, and Fleming[4] should be added in the polychromatic transmittance expression, i. e. Eq. (6). In this paper $\tau_i (i = 1, 2, \cdots, n)$ appeared later represents the polychromatic transmittances.

After adding bandwidth modification terms to Eq. (6), fitting calculations were carried out over and over again, and the terms with very small contribution to $\Delta\tau_i$ were removed progressively. In the meantime, calculations indicate that the polynomial coefficients vary with incident angle θ_i and can be represented by a power series of $\mu_i (= \cos\theta_i)$.

At last, a polychromatic τ_i model in a slant path from the top of the atmosphere ($i = 0$) to the i^{th} level is given with high accuracy as follows:

$$\begin{cases} \tau_0 = 1 \\ \tau_i = \tau_{i-1}\sum_{j=1}^{12} c_{ij}X_{ij} \\ c_{ij} = \sum_{k_\mu=1}^{6} C_{ijk_\mu}\mu_i'^{k_\mu-1} \\ c_{1,2} = c_{1,4} = c_{1,3} = {}_{1,9} = c_{1,11} = c_{1,12} = 0 \end{cases} \tag{7}$$

$(i = 1, 2, \cdots, n)$;

herein

$$X_{i,1} = 1 \quad , \tag{8}$$

$$X_{i,2} = \frac{1}{T_{i-1}} \quad , \tag{9}$$

$$X_{i,3} = \frac{1}{T_i'} = \frac{2}{T_{i-1} + T_i} \quad , \tag{10}$$

$$X_{i,4} = p_{i-1} \quad , \tag{11}$$

$$X_{i,5} = p_i' = \frac{p_{i-1} + p_i}{2} \quad , \tag{12}$$

$$X_{i,6} = X_{i,3}^2 \quad , \tag{13}$$

$$X_{i,7} = X_{i,4}^2 \quad , \tag{14}$$

$$X_{i,8} = X_{i,5}^2 \quad , \tag{15}$$

$$X_{i,9} = X_{i,2}X_{i,4} \quad , \tag{16}$$

$$X_{i,10} = X_{i,3}X_{i,5} \quad , \tag{17}$$

$$X_{i,11} = \Delta T_i^* = \sum_{m=1}^{i}(T_m - \tilde{T}_i)(\tilde{p}_m - \tilde{p}_{m-1}) \quad , \tag{18}$$

$$X_{i,12} = \Delta p_i^* = \sum_{m=1}^{i}(p_m - \tilde{p}_i)(\tilde{p}_m - \tilde{p}_{m-1}) \quad . \tag{19}$$

In the equations above, tilde represents the values listed in the USSA, asterisk represents bandwidth modification terms for polychromatic radiation and the coefficients c with values of zero show that the corresponding terms do not exist. For example, $c_{1,11} = c_{1,12} = 0$ represents that the 11^{th} and 12^{th} terms at the first level, i. e. $c_{1,11}X_{1,11}$ and $c_{1,12}X_{1,12}$, do not exist.

In Eq. (7) the subscript "0" represents the top of the atmosphere. In the calculations we take the pressure level with 0.01 hPa as a reference for the top of the atmosphere. In addition, the atmospheric refraction and the global curvature are taken into account.

In the purely physical retrieval method of the atmospheric pressure profile by means of O_2 absorption band near 762 nm, the mean atmospheric transmittances τ_i' from the top of the atmosphere to the i^{th} layer are usually needed too. τ_i' is regarded as a product of τ_i and a modification term $\Delta\tau_i'$:

$$\tau_i' = \tau_i\Delta\tau_i' \quad . \tag{20}$$

After fitting over and over again, we arrive at the polychromatic τ_i' model in a slant path from the top of the atmosphere to the i^{th} level as follows:

$$\begin{cases} \tau_1' = \tau_1\sum_{j=1}^{6} c_{1,j}'X_{1,j}' \\ \tau_i' = \tau_i\sum_{j=1}^{10} c_{ij}'X_{ij}' \quad (i = 2, 3, \cdots, n) \\ c_{ij}' = \sum_{k_\mu=1}^{4} C_{ijk_\mu}'\mu_{1,j}'^{k_\mu-1} \end{cases} \quad , \tag{21}$$

$(i = 1, 2, \cdots, n)$

herein

$$X_{1,1}' = 1 \quad , \tag{22}$$

$$X'_{1,2} = \frac{2}{T_0 + T_1} \quad , \tag{23}$$

$$X'_{1,3} = \frac{p_0 + p_1}{2} \quad , \tag{24}$$

$$X'_{1,4} = X'^{2}_{1,2} \quad , \tag{25}$$

$$X'_{1,5} = X'^{2}_{1,3} \quad , \tag{26}$$

$$X'_{1,6} = X'_{1,2}X'_{1,3} \quad ; \tag{27}$$

and

$$X'_{i,1} = 1 \quad , \tag{28}$$

$$X'_{i,2} = \frac{1}{T_i} \quad , \tag{29}$$

$$X'_{1,3} = \frac{4}{T_{i-1} + 3T_i} \quad , \tag{30}$$

$$X'_{i,4} = p_i \quad , \tag{31}$$

$$X'_{1,5} = \frac{p_{i-1} + 3p_i}{8} \quad , \tag{32}$$

$$X'_{i,6} = X'^{2}_{i,3} \quad , \tag{33}$$

$$X'_{i,7} = X'^{2}_{i,4} \quad , \tag{34}$$

$$X'_{i,8} = X'^{2}_{i,5} \quad , \tag{35}$$

$$X'_{i,9} = X'_{i,2}X'_{i,4} \quad , \tag{36}$$

$$X'_{i,10} = X'_{i,3}X'_{i,5} \quad , \tag{37}$$

$(i = 2,3,\cdots,n)$.

3 Determination of coefficients

The coefficients C_{ijk_μ} ($i=1,2,\cdots,n; j=1,2,\cdots,12; k_\mu=1,2,\cdots,6$) in Eq. (7) should be determined in two steps. At first, a set of atmospheric temperature-pressure profiles is given, whose number is much more than 12. The corresponding τ_i and τ_i' can be taken from the available τ_i database and the τ_i' database, respectively, and the predictors X_{ij} can be calculated according to Eq. (8) to Eq. (19) for each channel and each zenith angle θ_i ($i=1,2,\cdots,n$). Then, for each channel and each zenith angle the coefficients c_{ij} ($i=1,2,\cdots,n; j=1,2,\cdots,12$) are over-determined level by level from the first pressure level ($i=1$) to the lowest level ($i=n$) according to Eq. (7) by means of least square method.

As second step, for each channel all pairs(c_{ij}, μ_i') for same level ($i=1,2,\cdots,n$) and same term ($j=1,2,\cdots,12$) are used for over-determination of the polynomial coefficients C_{ijk_μ} ($k_\mu=1,2,\cdots,6$) according to Eq. (7) by means of the least mean method. The over-determination is carried out from the first level to the lower levels in turn.

Likewise, the polynomial coefficients C'_{ijk_μ} ($k_\mu = 1,2,3,4$) in Eq. (21) are also over-determined in two steps.

4 Calculation and results

We divide the atmosphere into 40 levels or layers ($n=40$), as same as specified by NOAA / NESDIS. In order to determine the coefficients in the atmospheric transmittance τ_i model and those in the mean atmospheric transmittance τ'_i model, at first, we should establish a database of τ_i ($i=1,2,\cdots,n$) and that of τ_i'($i=1,2,\cdots,n$) at different incident angles for 190 channels in the O_2 absorption band near 762 nm including some proximate window channels. Both τ_i and τ_i' are calculated according to an improved and accurate k-distribution method[17] by using the spectral line parameters in the HITRAN 1996. For the calculation we take the atmospheric refraction and the global curvature into account.

In this paper we take a bandwidth Δv of 1 cm^{-1} and consider 182 remote sounding channels in the O_2 absorption band near 762 nm (759.2 ~ 769.8 nm) with the central wave-numbers of 12990, 12991, $\cdots$, 13171 cm^{-1} and 8 window channels outside the absorption band with the central wave-numbers of 12930, 12940, 12950, 13180, 13190, 13195, 13205 and 13220 cm^{-1}. On the other hand, due to lack of measurements of the spectral response function of an interferometer spectrometer we assume a spectral response function for their 190 channels to be a parabola:

$$\Phi(v) = 1 - 2(v - v_0)^2$$

The used 26 incident angles outside the atmosphere are 0°, 10°, 20°, 26°, 30°, 33°, 36°, 38°, 40°, 42°, 44°, 46°, 47.5°, 49°, 50°, 51°, 52°, 53°, 54°, 55°, 56°, 57°, 57.9°, 58.7°, 59.4° and 60°. The fitting errors of τ_i in RMS sense calculated according to the improved k-distribution method in the range of $\theta = 0° - 60°$ are less than 3×10^{-6} relative to τ_i calculated according to the accurate line-by-line method (LBL) without any simplification[17].

Nine artificial atmospheric temperature profiles for determining the polynomial coefficients of the τ_i model and those of the τ_i' model are close to the profiles specified by McMillin and Fleming. In addition, we use also other 7 artificial atmospheric temperature profiles that are made by translation of the $\tilde{T}_i$($i=1,2,\cdots,40$) in USSA, namely $\tilde{T}_i$, $\tilde{T}_i\pm8K$, $\tilde{T}_i\pm16K$ and $\tilde{T}_i\pm24K$ ($i=1,2,\cdots,40$). On the other hand, we use the 10 atmospheric pressure profiles as follows:

$$\begin{cases} p_i = p_n \exp\left(- \sum_{k=i}^{n} \frac{g'_k M \Delta z_k}{R^* T'_k} \right) \\ (i = 1,2,\cdots,n-1) \\ p_n = \tilde{p}_{40}(1+t) \\ t = \pm 0.075, \pm 0.050, \pm 0.030, \pm 0.015, \pm 0.005 \end{cases} \quad . \quad (38)$$

At last, according to the τ_i model 190 × 40 = 7600 formulae are fitted by means of 190 ×26 ×16 × 10 ×40 = 31616000 τ_i data. According to these formulae $\tau_i(i=1,2,\cdots,n)$ can be calculated for all the 190 channels at arbitrary incident angles.

In order to check the accuracy of τ_i model effectively, we select two extreme atmospheric temperature profiles with very thick and very strong inverse temperature layers above the earth surface. Both the temperature profiles are highly close to those given by McMillin and Fleming[4]. The one is the extreme atmospheric temperature profile registered at the meteorological station Point Mugu, whereas another is artificial and named Abnormal profile. Calculations indicate that due to an accumulation effect the fitting error increases with depression of altitude, while it is insensitive to the incident angle. Then, the fitting errors (RMS) of τ_i and $[\tau_i - \tau_i(\text{real})]/\tau_i(\text{real})$ are calculated one by one according to 16 × 10 temperature-pressure models using 26 incident angles and 40 levels for each of all the 190 channels and further the total fitting errors for every channel are calculated. In addition, their maxima are also selected. At last, in order to describe the efficiency of the τ_i model, statistics are made for distribution of the number of channel appearing in different fitting error (RMS) ranges for both the two extreme atmospheric temperature profiles. The results are listed in Table 1.

As can be seen from Table 1, under the vast majority of circumstances the calculated values of τ_i are different from their real values in an RMS sense by less than 0.0001 (e_1) for all the 190 channels, whereas the relative errors (RMS)(e_2) are less than 0.0002. On the other hand, the maximal relative errors of e_1, i.e. e_3, are less than 0.0005, and that of e_2, i.e. e_4, closes to 0.018.

At last, the fitting errors (RMS) and their maxima for Point Mugu atmospheric temperature profile and Abnormal profile in the τ_i model given in this paper were compared with those in other τ_i models, as shown in Table 2.

Table 1 Distributions of number of channel with different fitting errors (RMS) for τ_i and $[\tau_i - \tau_i(\text{real})]/\tau_i(\text{real})$

表1 τ_i 和 $[\tau_i - \tau_i(\text{real})]/\tau_i(\text{real})$ 拟合误差在不同精度范围内的通道个数

fitting errors	$3\cdot10^{-3}$ – $1\cdot10^{-3}$	$1\cdot10^{-3}$ – $5\cdot10^{-3}$	$5\cdot10^{-3}$ – $2\cdot10^{-4}$	$2\cdot10^{-4}$ – $1\cdot10^{-4}$	$1\cdot10^{-4}$ – $1\cdot10^{-5}$	$<1\cdot10^{-5}$
Point Mugu profile:						
e_1	0	0	5	13	114	58
e_2	0	2	16	49	66	57
e_3	3	5	72	45	59	6
e_4	24	50	33	18	60	5
Abnormal profile:						
e_1	0	0	6	5	105	74
e_2	0	2	23	30	65	70
e_3	2	3	84	28	62	11
e_4	50	29	32	11	57	11

*e_1: Number of channels with fitting errors of τ_i in a given range,
e_2: Number of channels with fitting errors of $[\tau_i - \tau_i(\text{real})]/\tau_i(\text{real})$ in a given range,
e_3: Number of channels with maximal fitting errors of τ_i in a given range,
e_4: Number of channels with maximal fitting errors of $[\tau_i - \tau_i(\text{real})]/\tau_i(\text{real})$ in a given range

The results indicate that the accuracy of the τ_i model given in this paper is much higher than those of

Table 2 Comparisons among fitting errors (RMS) (left) and maximal fitting errors (right) of different atmospheric transmittance models

表2 不同大气透射模型拟合误差(左)与最大拟合误差(右)之比较

Range of Errors		$<1\cdot10^{-4}$	$1\cdot10^{-3}$ – $3\cdot10^{-3}$		$<1\cdot10^{-4}$	$1\cdot10^{-3}$ – $3\cdot10^{-3}$
This paper	number of channel	351/364	0/364	number of channel	138/364	5/364
Ref. [4]	in range	1/12	1/12	in range	0/12	9/12
Ref. [5]	of fitting	5/60	11/60	of	0/60	50/60
	error/	1/31	10/31	maximal	0/31	21/31
Ref. [6]	total number of channel		(.0046 – .0090: 2/31)	fitting error/ total number of channel		(.0031 – .0345: 9/31)
Ref. [11]			RMS = 0.0101 (for all the 48 channels in band 12850 – 13085 cm^{-1})			

Notes: Reference [7, 8]: The fitting errors (RMS) and the maximal fitting errors are not given.

models given in other studies.

Likewise, according to the τ_i' model 7600 formulae are fitted by means of 31616000 τ_i' data as well. The results are given in Table 3. As can be seen from Table 3, under the vast majority of circumstances the calculated values of τ_i' for all the channels are different from their real values in an RMS sense by less than 0.0005 (e'_1), whereas their relative errors (RMS), $e_2' = [\tau_i' - \tau_i'(\text{real})]/\tau_i'(\text{real})$, are less than 0.001. On the other hand, the maximal errors of e_1', i. e. e_3', are less than 0.003, and that of e_2', i. e. e_4', can be as large as 0.019. Ten channels with e_4' larger than 0.01 are not used in the retrieval procedure of atmospheric pressure.

Table 3 Distributions of number of channel with different fitting errors (RMS) for τ_i' and $[\tau_i' - \tau_i'(\text{real})]/\tau_i'(\text{real})$

表3 τ_i'和$[\tau_i' - \tau_i'(\text{real})]/\tau_i'(\text{real})$拟合误差在不同精度范围内的通道个数

fitting errors	$1.9\cdot10^{-2}$ – $3\cdot10^{-3}$	$3\cdot10^{-3}$ – $1\cdot10^{-3}$	$1\cdot10^{-3}$ – $5\cdot10^{-4}$	$5\cdot10^{-4}$ – $2\cdot10^{-4}$	$2\cdot10^{-4}$ – $1\cdot10^{-4}$	$1\cdot10^{-4}$ – $1\cdot10^{-5}$	$<1\cdot10^{-5}$
			Point Mugu profile:				
e_1'	0	2	16	5	18	64	45
e_2'	0	21	37	19	13	56	44
e_3'	15	55	17	23	21	57	2
e_4'	59	29	10	16	19	55	2
			Abnormal profile:				
e_1'	0	2	21	47	17	64	39
e_2'	0	28	35	25	9	54	39
e_3'	8	62	36	19	19	45	1
e_4'	66	33	14	14	17	45	1

*e_1': Number of channels with fitting errors of τ_i' in a given range,
e_2': Number of channels with fitting errors of $[\tau_i' - \tau_i'(\text{real})]/\tau_i'(\text{real})$ in a given range,
e_3': Number of channels with maximal fitting errors of τ_i' in a given range,
e_4': Number of channels with maximal fitting errors of $[\tau_i' - \tau_i'(\text{real})]/\tau_i'(\text{real})$ in a given range.

It is noted that the probability for the appearing of large maximal errors is actually very small. Furthermore, two extreme profiles, i. e. Point Mugu profile and Abnormal profile, are used for accuracy tests of the τ_i models. In other words, the accuracy tests are carried out under most severe conditions. Therefore, the atmospheric transmittances and the mean atmospheric transmittances calculated for actual atmosphere is more accurate than those listed in the Table 1 and Table 3.

5 Method of removing interval error of atmospheric transmittance

In the iterative procedure the atmospheric pressure equation in differential form, Eq. (38), can be used many times for retrieval and forward calculation. However, it is not accurate enough to divide the atmosphere into 40 levels or layers to calculate the pressure at the i^{th} level, $p_i (i = 1, 2, \cdots, 40)$, so an interval error of p_i, i. e. $e(i)p_i$, would be introduced. Herein $e(i)$ represents the relative interval error of p_i:

$$e(i) = \delta p_i / p_i = [p_i(\text{with interval error}) - p_i]/p_i \quad . \tag{39}$$

However, the interval errors of $\tau_i (i = 1, 2, \cdots, n)$ caused by dividing the atmosphere can be corrected, if priori values of the interval error of $p_i (i = 1, 2, \cdots, n)$ are calculated in advance according to $T(p)$.

Considering the optical depth $L_i = k_i' q_i' \rho_i' \mu_i' \Delta z_i$ of an atmospheric layer and the equation of state gives

$$\Delta\tau_i = \frac{\tau_i}{\tau_{i-1}} = \exp(-L_i) = \exp\left(-k_i' q_i' \frac{M p_i' \mu_i'}{R^* T_i'} \Delta z_i\right) \quad , \tag{40}$$

in which p_i' and T_i' are averages of the p and T at the $(i-1)^{th}$ and the i^{th} levels, respectively. Considering Eqs. (38) to (40), we arrive at the interval error of $\Delta\tau_i$ caused by the interval error of p_i:

$$\delta(\Delta\tau_i) = \delta\left(\frac{\tau_i}{\tau_{i-1}}\right) = \frac{\tau_i}{\tau_{i-1}} \cdot \frac{1}{p_i'} \ln \frac{\tau_i}{\tau_{i-1}} \cdot \frac{\delta p_{i-1} + \delta p_i}{2} = \frac{\tau_i}{\tau_{i-1}} \ln \frac{\tau_i}{\tau_{i-1}} \cdot \frac{p_{i-1} e(i-1) + p_i e(i)}{2p_i'} \quad . \tag{41}$$

The correction is carried out sequentially as τ_i is being calculated from the top of the atmosphere downwards, so τ_{i-1} in Eq. (41) is a corrected value $\hat{\tau}_{i-1}$. Then, we arrive at the relative interval error of atmospheric transmittance:

$$\frac{\delta\tau_i}{\tau_i} = \ln \frac{\tau_i}{\hat{\tau}_{i-1}} \cdot \frac{p_{i-1} e(i-1) + p_i e(i)}{2p_i'} \quad . \tag{42}$$

Now, p_i calculated by means of Eq. (38) according to the 590 fine altitude intervals specified in the USSA under circumstance of given $T(p)$ are regarded as the real atmospheric pressure profile and p_i (with interval error) is also calculated for the 40 atmospheric pressure levels specified by NOAA / NESDIS, so $e(i)$ can be determined according to Eq. (39). Consequently, $\delta\tau_i/\tau_i$ is calculated according to Eq. (42). At last, we arrive at the correction value $\delta\tau_i$ and the correct value $\tau_i + \delta\tau_i$ of the atmospheric transmittance.

6 Conclusions

In this paper absorption coefficient is regarded, based on physical essence, as a function of atmospheric pressure and reciprocal of atmospheric temperature. As a result, under vast majority of circumstances the fitting errors (RMS) of atmospheric transmittance τ_i model and their maximal errors are less than 0.0001 and 0.0005, respectively. The accuracy of the τ_i model given in this paper is much higher than those of other models.

In the meantime, a mean atmospheric transmittance τ_i' model is also given in this paper. The fitting errors (RMS) and their maxima of the mean transmittance τ_i' model are less than 0.0005 and 0.003, respectively, and also considerable accurate.

Both the τ_i model and the τ_i' model were applied to the study on remote sounding of atmospheric pressure profile from space[13-15].

In this paper, a method for removing internal errors of atmospheric tremsmittance is given, but temporarily we can not give a proof for the efficiency of the correction equation, because the accurate τ_i ($i = 1, 2, \cdots, 40$) model calculated for the atmosphere divided into 590 very fine intervals is not available. On the other hand, the interval errors of τ_i caused by those of $T(p_i)$ are not considered in this paper.

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