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非线性散射介质内辐射传递的积分矩方法

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摘要:提出了求解非线性散射介质内辐射传递的积分矩方法. 将辐射传递方程中散射相函数的积分项转化为辐射强度各阶矩的线性组合. 散射相函数为勒让德多项式展开形式,辐射强度矩的最高阶数与散射相函数的展开项数相同. 将原本复杂的积分微分方程转化为微分方程,通过积分法求解此方程. 积分矩方法不需要对立体角进行离散,不会引起射线效应. 积分矩方法的计算结果与其他数值方法符合得很好,具有很高的计算精度. 同时,在较少计算网格数的条件下,仍然可以得到较为满意的结果.

关 键 词:辐射传递;非线性散射;积分矩方法
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Integral moment method for radiative transfer in nonlinear scattering medium

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Abstract: The integral moment method was proposed to solve radiative transfer in nonlinear scattering medium. In this method, the integral term of the scattering phase function was deduced to linear combinations of radiative intensity moments. The scattering phase function was expressed as a truncated Legendre series and the highest order of radiative intensity moments was equal to the approximation order of the scattering phase function. Then, the integro-differential equation was reduced to differential equation, which is solved by integral method. Since it is not required to discretize the solid angle, the integral moment method will not suffer ray effects. The results by the present method are in good agreement with those in references and the method has a high accuracy. Furthermore, satisfied results also can be obtained even in the case of fewer computational meshes.

Key words: radiative transfer; nonlinear scattering; integral moment method
PACS: 44.40. +a, 44.05. +e

引言

半透明介质内的辐射传递研究在目标红外特性、遥感探测、红外加热、耐热涂层以及太阳能应用等工程领域有重要的应用背景. 对于含粒子的介质,其粒子的非线性散射对于红外辐射特性计算有重要的影响[1].

近年来,已发展了多种求解半透明介质内辐射传递的数值方法,如蒙特卡洛法、离散传递法、离散坐标法[2]、有限元法[3]. 但是,对于高度非线性散射介质内的辐射传递问题的求解,上述方法的相关应用并不多见. Modest[4]通过积分方法给出了一维半透明线性散射灰介质的精确解,对于非线性散射介质的积分方法,国内外的报道较少.

提出了求解非线性散射介质内辐射传递的积分矩方法. 该方法将辐射传递方程中散射相函数的积分项转化为辐射强度各阶矩的线性组合,并且辐射强度矩的最高阶数与散射相函数按勒让德多项式的展开阶数相同. 该方法将原本复杂的积分微分方程

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转化为微分方程，然后通过积分法求解此方程. 为了减少计算量和提高计算速度，考察了计算网格数对数值结果的影响.

1 求解辐射传递方程的积分矩方法

对于一维吸收、发射、散射灰介质，辐射传递方程(RTE)为

$$\begin{aligned}&\cos\theta \mathrm{d}I(\tau,\theta)/\mathrm{d}\tau + I(\tau,\theta)\\&= (1-\omega)I_b(\tau)\\&+(\omega/2)\int_{\theta'=0}^{\pi} I(\tau,\theta')\Phi(\theta,\theta')\sin\theta'\mathrm{d}\theta'\end{aligned} \quad, \qquad (1)$$

其中 θ 为天顶角方向，τ 为光学厚度，$I_b(\tau)$ 为黑体辐射强度，ω 为散射反照率，$\Phi(\theta,\theta')$ 为散射相函数. 对于不透明、漫反射灰壁面，边界条件为

$$I(0) = I_b(0) - (1-\varepsilon_0)q(0)/\pi\varepsilon_0 \quad, \qquad (2a)$$

$$I(\tau_d) = I_b(\tau_d) + (1-\varepsilon_d)q(\tau_d)/\pi\varepsilon_d \quad, \qquad (2b)$$

其中 $q(0)$，$q(\tau_d)$ 分别为两壁面处的辐射热流密度，ε_0，ε_d 分别为两壁面处的发射率.

散射相函数可按勒让德多项式展开，最终得到关于 $\cos\theta$ 和 $\cos\theta'$ 的二元多项式

$$\begin{aligned}\Phi(\theta,\theta') &= \sum_{i=0}^{M} g_i P_i(\cos\theta)P_i(\cos\theta')\\&= \sum_{i=0}^{M}\sum_{j=0}^{M} A_{ij}\cos^i\theta\cos^j\theta'\end{aligned} \quad, \qquad (3)$$

其中 g_i 为散射相函数的不对称因子，A_{ij} 为散射相函数关于 $\cos\theta$ 和 $\cos\theta'$ 的二元多项式系数.

定义辐射强度的各阶矩为

$$\begin{aligned}&I_m(\tau) = 2\pi\int_{\theta=0}^{\pi} I(\tau,\theta)\cos^m\theta\sin\theta\mathrm{d}\theta,\\&m = 0,1,\cdots,M\end{aligned} \quad, \qquad (4)$$

其中 $I_0(\tau)$ 为介质的投射辐射 $G(\tau)$，$I_1(\tau)$ 为介质的辐射热流密度 $q(\tau)$.

将式(3)和式(4)代入式(1)，得

$$\begin{aligned}&\cos\theta\mathrm{d}I(\tau,\theta)/\mathrm{d}\tau + I(\tau,\theta)\\&= (1-\omega)I_b(\tau) +\\&(\omega/4\pi)\sum_{i=0}^{M}\sum_{j=0}^{M} A_{ij}I_j(\tau)\cos^i\theta\end{aligned} \quad. \qquad (5)$$

由式(5)可以看出，辐射强度 $I(\tau,\theta)$ 只与其前 $M+1$ 阶矩 $(0,1,\cdots,M)$ 有关，而与此后的各阶矩无关.

对式(5)进行积分，得

$$\begin{aligned}I^+(\tau,\theta) &= I(0)\exp\{-\tau/\cos\theta\} + (1-\omega)\int_{\tau'=0}^{\tau} I_b(\tau')\exp\{-(\tau-\tau')/\cos\theta\}/\cos\theta\mathrm{d}\tau'\\&+(\omega/4\pi)\sum_{i=0}^{M}\sum_{j=0}^{M} A_{ij}\int_{\tau'=0}^{\tau} I_j(\tau')\cos^{i-1}\theta\exp\{-(\tau-\tau')/\cos\theta\}\mathrm{d}\tau'\end{aligned} \quad, \qquad (6a)$$

$0 < \theta < \pi/2$

$$\begin{aligned}I^-(\tau,\theta) &= I(\tau_d)\exp\{(\tau_d-\tau)/\cos\theta\} - (1-\omega)\int_{\tau'=\tau}^{\tau_d} I_b(\tau')\exp\{(\tau'-\tau)/\cos\theta\}/\cos\theta\mathrm{d}\tau'\\&-(\omega/4\pi)\sum_{i=0}^{M}\sum_{j=0}^{M} A_{ij}\int_{\tau'=\tau}^{\tau_d} I_j(\tau')\cos^{i-1}\theta\exp\{(\tau'-\tau)/\cos\theta\}\mathrm{d}\tau'\end{aligned} \quad. \qquad (6b)$$

$\pi/2 < \theta < \pi$

将式(6)代入式(4)，并令 $\mu = |\cos\theta|$，得

$$\begin{aligned}I_m(\tau) &= 2\pi\Big[\int_{\mu=0}^{1} I(0)\exp\{-\tau/\mu\}\mu^m\mathrm{d}\mu + (-1)^m\int_{\mu=0}^{1} I(\tau_d)\exp\{-(\tau_d-\tau)/\mu\}\mu^m\mathrm{d}\mu\\&+(1-\omega)\int_{\tau'=0}^{\tau}\int_{\mu=0}^{1} I_b(\tau')\exp\{-(\tau-\tau')/\mu\}\mu^{m-1}\mathrm{d}\mu\mathrm{d}\tau'\\&+(-1)^m(1-\omega)\int_{\tau'=\tau}^{\tau_d}\int_{\mu=0}^{1} I_b(\tau')\exp\{-(\tau'-\tau)/\mu\}\mu^{m-1}\mathrm{d}\mu\mathrm{d}\tau'\\&+(\omega/4\pi)\sum_{i=0}^{M}\sum_{j=0}^{M} A_{ij}\int_{\tau'=0}^{\tau}\int_{\mu=0}^{1} I_j(\tau')\exp\{-(\tau-\tau')/\mu\}\mu^{i+m-1}\mathrm{d}\mu\mathrm{d}\tau'\\&+(\omega/4\pi)\sum_{i=0}^{M}\sum_{j=0}^{M}(-1)^{i+m}A_{ij}\int_{\tau'=\tau}^{\tau_d}\int_{\mu=0}^{1} I_j(\tau')\exp\{-(\tau'-\tau)/\mu\}\mu^{i+m-1}\mathrm{d}\mu\mathrm{d}\tau'\Big]\end{aligned} \quad, \qquad (7)$$

对于边界条件

$$\begin{aligned}q(0) = I_1(0) &= 2\pi\Big[\int_{\mu=0}^{1} I(0)\mu\mathrm{d}\mu - \int_{\mu=0}^{1} I(\tau_d)\exp\{-\tau_d/\mu\}\mu\mathrm{d}\mu\\&-(1-\omega)\int_{\tau'=0}^{\tau_d}\int_{\mu=0}^{1} I_b(\tau')\exp\{-\tau'/\mu\}\mathrm{d}\mu\mathrm{d}\tau'\\&-(\omega/4\pi)\sum_{i=0}^{M}\sum_{j=0}^{M}(-1)^i A_{ij}\int_{\tau'=0}^{\tau_d}\int_{\mu=0}^{1} I_j(\tau')\exp\{-\tau'/\mu\}\mu^i\mathrm{d}\mu\mathrm{d}\tau'\Big]\end{aligned} \quad, \qquad (8a)$$

$$
\begin{aligned}
q(\tau_d) = I_1(\tau_d) = 2\pi\Big[&\int_{\mu=0}^{1} I(0)\exp\{-\tau_d/\mu\}\mu d\mu - \int_{\mu=0}^{1} I(\tau_d)\mu d\mu \\
&+ (1-\omega)\int_{\tau'=0}^{\tau_d}\int_{\mu=0}^{1} I_b(\tau')\exp\{-(\tau_d-\tau')/\mu\} d\mu d\tau' \\
&+ (\omega/4\pi)\sum_{i=0}^{M}\sum_{j=0}^{M} A_{ij}\int_{\tau'=0}^{\tau_d}\int_{\mu=0}^{1} I_j(\tau')\exp\{-(\tau_d-\tau')/\mu\}\mu^i d\mu d\tau'\Big]
\end{aligned} \quad . \tag{8b}
$$

将介质离散为 N 个微层(不包括两个壁面),每微层中心处的光学厚度为 $\tau_n(n=1,\cdots,N)$,每微层内的 $T, I_k(k=0,1,\cdots,M)$ 相等,微层中心处分别为 T_n, I_{kn},如图 1 所示. 利用指数积分函数 $E_n(x)$ 及其递推关系,可将式(7)和式(8)化为线性代数方程组

$$
\begin{aligned}
I_{mn} = &2\pi I(0)E_{m+2}(\tau_n) + (-1)^m 2\pi I(\tau_d)E_{m+2}(\tau_d-\tau_n) \\
&+ 2\pi(1-\omega)\Big\{\sum_{i=1}^{n-1} I_b(\tau_i)[E_{m+2}(\tau_n-\bar{\tau}_{i+1}) - E_{m+2}(\tau_n-\bar{\tau}_i)] \\
&+ I_b(\tau_n)[E_{m+2}(0) - E_{m+2}(\tau_n-\bar{\tau}_n)]\Big\} \\
&+ 2\pi(-1)^m(1-\omega)\Big\{\sum_{i=n+1}^{N} I_b(\tau_i)[E_{m+2}(\bar{\tau}_i-\tau_n) - E_{m+2}(\bar{\tau}_{i+1}-\tau_n)] \\
&+ I_b(\tau_n)[E_{m+2}(0) - E_{m+2}(\bar{\tau}_{n+1}-\tau_n)]\Big\} \\
&+ (\omega/2)\sum_{i=0}^{M}\sum_{j=0}^{M} A_{ij}\Big\{\sum_{k=1}^{n-1} I_j(\tau_k)[E_{i+m+2}(\tau_n-\bar{\tau}_{k+1}) - E_{i+m+2}(\tau_n-\bar{\tau}_k)] \\
&+ I_j(\tau_n)[E_{i+m+2}(0) - E_{i+m+2}(\tau_n-\bar{\tau}_n)]\Big\} \\
&+ (\omega/2)\sum_{i=0}^{M}\sum_{j=0}^{M}(-1)^{i+m} A_{ij}\Big\{\sum_{k=n+1}^{N} I_j(\tau_k)[E_{i+m+2}(\bar{\tau}_k-\tau_n) - E_{i+m+2}(\bar{\tau}_{k+1}-\tau_n)] \\
&+ I_j(\tau_n)[E_{i+m+2}(0) - E_{i+m+2}(\bar{\tau}_{n+1}-\tau_n)]\Big\}
\end{aligned} \quad , \tag{9}
$$

$$
\begin{aligned}
q(0) = &\pi I(0) - 2\pi I(\tau_d)E_3(\tau_d) - 2\pi(1-\omega)\sum_{i=1}^{N} I_b(\tau_i)[E_3(\bar{\tau}_i) - E_3(\bar{\tau}_{i+1})] \\
&- (\omega/2)\sum_{i=0}^{M}\sum_{j=0}^{M}(-1)^i A_{ij}\sum_{k=1}^{N} I_j(\tau_k)[E_{i+3}(\bar{\tau}_k) - E_{i+3}(\bar{\tau}_{k+1})]
\end{aligned} \quad , \tag{10a}
$$

$$
\begin{aligned}
q(\tau_d) = &2\pi I(0)E_3(\tau_d) - \pi I(\tau_d) + 2\pi(1-\omega)\sum_{i=1}^{N} I_b(\tau_i)[E_3(\tau_d-\bar{\tau}_{i+1}) - E_3(\tau_d-\bar{\tau}_i)] \\
&+ (\omega/2)\sum_{i=0}^{M}\sum_{j=0}^{M} A_{ij}\sum_{k=1}^{N} I_j(\tau_k)[E_{i+3}(\tau_d-\bar{\tau}_{k+1}) - E_{i+3}(\tau_d-\bar{\tau}_k)]
\end{aligned} \quad . \tag{10b}
$$

联立求解式(2)、式(9)和式(10),可得 I_{mn}.

2 计算结果与分析

2.1 非线性散射介质内辐射传递的数值验证

考虑一维非线性各向异性散射灰介质内的辐射传递,两壁面假设为黑体,温度分别为 $T_0=0$, $T_d=1000$. 定义无量纲反射辐射热流 $\psi = 1 - q_0/\sigma T_d^4$,其中 q_0 为界面 1 处的投射辐射热流. 介质的散射反照率 $\omega=1$,考虑以下非线性散射相函数,计算不同光学厚度 τ_d 时的 ψ. 计算网格数 $N=1000$.

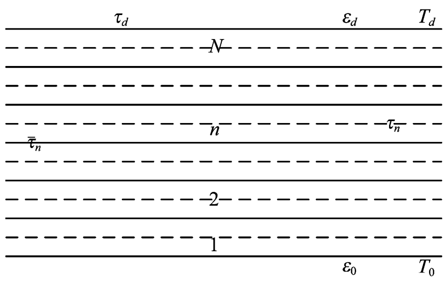

图 1 介质离散化

Fig. 1 Discretization of the medium

$$(1)\ \Phi_1 = 1 + 0.5P_2(\cos\Theta) \quad , \tag{11a}$$

$$(2)\ \Phi_2 = 1 + P_1(\cos\Theta) + 0.5P_2(\cos\Theta) \quad , \tag{11b}$$

Busbridge、Orchard[2] 与积分矩方法的计算结果见表 1. 由表 1 可见,积分矩方法的计算结果与上述文献中的计算结果符合得很好,最大相对误差小于 0.01%.

表 1 非线性散射介质无量纲反射辐射热流

Table 1 Dimensionless reflective radiative heat flux for nonlinear scattering medium

τ_d	Φ_1		Φ_2	
	Orchard	IM	Busbridge	IM
1	0.4468	0.446775	0.3580	0.358048
2	0.6101	0.610121	0.5157	0.515715
3	0.6985	0.698477	0.6104	0.610364
4	0.7541	0.754112	0.6739	0.673939
5	0.7924	0.792402	0.7197	0.719653
6	0.8204	0.820370	0.7541	0.754120
7	0.8417	0.841696	0.7810	0.781039
8	0.8585	0.858495	0.8026	0.802645
9	0.8721	0.872071	0.8204	0.820369
10	0.8833	0.883270	0.8352	0.835172

为验证积分矩方法对高阶非线性散射相函数的适应性,考虑以下散射相函数:

前向散射相函数 $F2$

$$\begin{aligned}F2(\cos\Theta) = 1 + 2.00917P_1(\cos\Theta) + 1.56339P_2(\cos\Theta) \\ + 0.67407P_3(\cos\Theta) + 0.22215P_4(\cos\Theta) \\ + 0.04725P_5(\cos\Theta) + 0.00671P_6(\cos\Theta) \\ + 0.00068P_7(\cos\Theta) + 0.00005P_8(\cos\Theta)\end{aligned} \quad . \quad (12a)$$

后向散射相函数 $B1$

$$\begin{aligned}B1(\cos\Theta) = 1 - 0.56524P_1(\cos\Theta) + 0.29783P_2(\cos\Theta) \\ + 0.08571P_3(\cos\Theta) + 0.01003P_4(\cos\Theta) \\ + 0.00063P_5(\cos\Theta)\end{aligned} \quad . \quad (12b)$$

计算条件为:光学厚度 $\tau_d = 1$,计算网格数为 $N = 1000$,其他计算条件同上例. 积分矩方法与 P_N 法[5](网格数10000,近似阶数31)的比较结果见表2. 由表2可见,两种方法的计算结果非常接近,相对误差不超过0.01%.

表2 高阶非线性散射介质无量纲反射辐射热流
Table 2 Dimensionless reflective radiative heat flux for high order nonlinear scattering medium

$F2$		$B1$	
P_N	IM	P_N	IM
0.233837	0.233836	0.486803	0.486788

2.2 计算网格数对数值结果的影响

为了减少计算量和提高计算速度,考察计算网格数对计算精度的影响,以非线性散射相函数 Φ_3 为例,计算不同光学厚度下的无量纲反射辐射热流,计算不同网格数下的曲线见图2.

$$\Phi_3 = 1 + 1.5P_1(\cos\Theta) + 0.5P_2(\cos\Theta) \quad . \quad (13)$$

由图2可见,较少的网格数($N = 8$),积分矩方法也可以得到较好的结果,和网格数 $N = 1000$ 的结果比较,最大相对误差为2.4%,可见该方法可明显减少计算量,计算速度也会得到较大提高.

3 结论

提出了求解非线性散射介质内辐射传递的积分矩方法. 该方法将辐射传递方程中散射相函数的积分项转化为辐射强度各阶矩的线性组合,并且辐射强度矩的最高阶数与散射相函数按勒让德多项式的展开阶数相同. 根据理论推导,辐射强度只与散射相函数展开阶数之前的各阶矩有关,而与此后的各阶矩无关. 积分矩方法将原本复杂的积分微分方程转化为微分方程,最后通过积分法求解该方程.

积分矩方法不需要对立体角进行离散,因此,该方法不会引起射线效应. 通过与其他数值方法比较,考虑几种非线性散射相函数,结果显示积分矩方法求解非线性散射介质内辐射传递问题有很好的计算精度;对于高阶非线性散射相函数,该方法依然和其他方法符合得很好. 同时,该方法在较少网格数的条件下,计算量减少,计算速度提高,并且得到较为满意的结果.

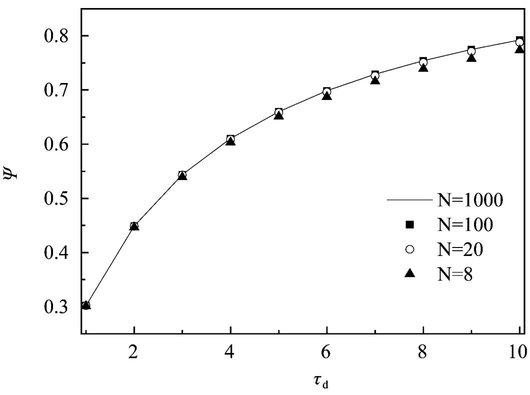

图2 计算网格数对非线性散射无量纲反射辐射热流的影响
Fig. 2 The effects of computational meshes number on the dimensionless reflective radiative heat flux for nonlinear scattering

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